Information-theoretic approach to sensor selection

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1 Motivation

It is important to be able to explore and estimate target state by using distributed sensing. For such sensing tasks not all measurements are equally informative. Therefore, for large sensor networks the relevant data can either be extracted by removing correlated measurements in post statistical analysis, or by limiting the number of selected sensors. The first method is computationally intensive (especially for the case of a moving target) when fast successive measurements are desired. Therefore, a sensing task should employ more sensors than necessary, and only use the ones that provide the most information. From an information-theoretic perspective each sensor is tasked with observing the target and reducing the ambiguity of target state. The information gain associated to each sensor can be different if the system is anisotropic (Fig. 1D). Therefore, repeatedly selecting sensors that are most informative reduces the overall uncertainty of the target parameters. This allows us to represent the problem as choosing k sensors among m possible sensors to minimize the error of estimating target state.

Several heuristic have been proposed for optimal selection of sensors. These include genetic algorithms [1], mutual information [2,3], information gain maximization [4,5]. Algorithms based on these heuristics rely on selecting sensors that are optimal in the next given configuration, and therefore can only locally optimize. We propose the use of ergodic exploration of distributed information (EEDI) [6], which compares the statistics of selected sensors to a map of expected information density.

We test the proposed selection algorithm on a system that uses electrosense with an array like distribution of sensors between the emitters (Fig 1A), modeled after the weakly electric fish *Apteronotus albifrons*. We show in simulation that given an object by iteratively selecting sensors we can localize a sphere of known radius and z-coordinate in an x,y workspace.

2 Sensor Selection Heuristic and Control

An array of sensors allows us to mimic the sensor distribution of the electric fish. Due to fourth power signal falloff, for distant objects, signals at adjacent electrodes are strongly correlated. However, for nearby objects, local sensor density is important, and the peripheral array can largely be ignored. This is exemplified in Fig. 1C which shows the perspective of each sensor for all possible target locations. Therefore, sensor placement and choice determines the amount of information that can be obtained about target parameters.

Two conductors act as voltage emitting electrodes, and in between them there are sixteen evenly distributed sensors in a grid structure, with high input impedance to neglect flowing currents. Out of the sixteen sensors, four are chosen with predefined constraints, which limits the possible number sensor of configurations.

2.1 Idealized Model

We considered an idealized 3D model for the observed voltage perturbation created by a conductive sphere of known radius a (Eq. 2). If we let $\mathbf{E_f}$ be the electric field vector at the location of the target, $\mathbf{r} \in \mathbb{R}^3$ represent the target-centered relative coordinates, then we can represent the change in potential δV (mV) as:

$$M = \frac{(\sigma_s + i\omega\varepsilon_s) - (\sigma_m + i\omega\varepsilon_m)}{(\sigma_s + i\omega\varepsilon_s) + 2(\sigma_m + i\omega\varepsilon_m)} \quad (1)$$

$$\delta V(\mathbf{r}) = \frac{a^3 M}{|r|^3}\mathbf{E_f}\cdot\mathbf{r} \quad (2)$$

where $\sigma_s,\ \varepsilon_s$ is the conductivity and permittivity of the sphere, respectively. Similarly, $\sigma_m,\ \varepsilon_m$ refers to the conductivity and permittivity of the medium, respectively.

2.2 Sensor Selection Algorithm

For selecting sensors we use ergodic exploration of distributed information algorithm, which compares the statistics of a search trajectory to the expected information density [6]. A trajectory is generated such that the majority of the time is spent in high information locations. This also allows for the exploration of regions that have low information content. In the case of incorrect expected information density, methods that employ information maximization are more likely to fail since instead of exploration they use exploitation. By combining both search strategies we are able to use a robust sensor selection algorithm while disambiguating the localization problem.

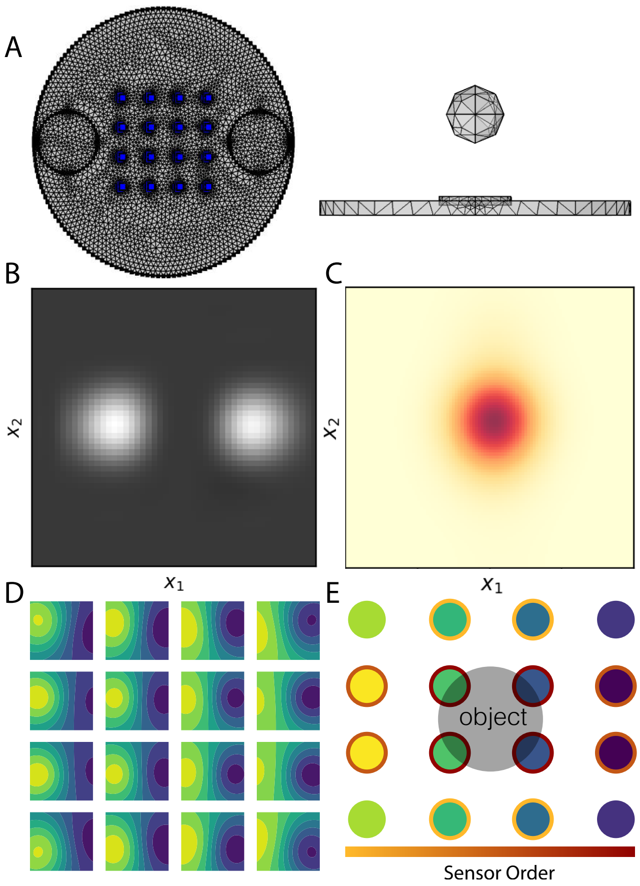

Figure 1: **A** The simulated array and object. **B, C** Expected information density and probability density function of a target placed at the center with low variance. **D** Sensor measurements for all possible object locations in a gridded x, y plane. **E** Iteratively chosen sensors and the measurements from each sensor. Light orange corresponds to the first set and Dark red corresponds to the last set.

For the given electrosensory array we assume a conductive sphere of radius 5 mm located at $z = 3$ cm. An initial sensor location and joint probability distribution $p(\Theta)$ of parameters Θ are chosen. If no prior knowledge about target state exists, $p(\Theta)$ is initialized to be uniform on a bounded domain. Bayesian filter is used to update the target state belief based on sensor measurements, ideal model, and noise estimates. In order to calculate the expected information density (EID) we calculate the expected Fisher information matrix, which is the expected value of the Fisher information with respect to the joint belief $p(\Theta)$ for pairs of parameters. We set the EID to be the determinant of the expected Fisher information matrix, based on the D-optimality metric [7]. Fig. 1C shows the belief ($p(\Theta)$) and EID for a target located at the center for low variance.

Ergodic trajectory [6] is calculated without the use of any kinematics for the sensor selection problem. The generated trajectory is then mapped onto to k available sensors. The k sensors are closest (Euclidean metric) to the trajectory, most informative (based on the EID), and unconstrained. Measurements collected by these sensors are then used to update the belief $p(\Theta)$ and EID, which are used to generate the next trajectory. The algorithm terminates when the norm of the standard deviation of the estimate falls below 0.03. Fig. 1D shows and example of sensor choices and measurements given a target centered at $x, y = (0, 0)$

3 Discussion

For a sensing task, not all measurements are equally informative. In the case of electrosense, due to fourth power signal falloff with distance, each object location and geometry will have a different set of most informative electrodes. Here we provide an algorithm which can be used for either sensor placement or sensor selection, to optimize both exploitation of information gain, and exploration of workspace. We use an electrosensory system to test the effectiveness of the algorithm. Under high variance, we show that we are able to localize the object with successive sensor measurements.

References

[1] Gardner, J. W., P. Boilot, and E. L. Hines. "Enhancing electronic nose performance by sensor selection using a new integer-based genetic algorithm approach." *Sensors and Actuators B: Chemical* 106.1 (2005): 114-121.

[2] J. Toh and S. Sukkarieh, A Bayesian formulation for the prioritized search of moving objects, in IEEE Int. Conf. on Robotics and Automation (ICRA), 2006, pp. 219224

[3] J. Denzler and C. Brown, Information theoretic sensor data selection for active object recognition and state estimation, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 24, no. 2, pp. 145157, 2002

[4] G. A. Hollinger, B. Englot, F. S. Hover, U. Mitra, and G. S. Sukhatme, Active planning for underwater inspection and the benefit of adaptivity, *International Journal of Robotics Research*, vol. 32, no. 1, pp. 318, 2013

[5] G. Zhang, S. Ferrari, and M. Qian, An information roadmap method for robotic sensor path planning, *Journal of Intelligent and Robotic Systems*, vol. 56, no. 1-2, pp. 6998, 2009.

[6] L. M. Miller, Y. Silverman, M. A. MacIver, and T. D. Murphey. Ergodic Exploration of Distributed Information. *IEEE Transactions on Robotics*, 32(1):3652, 2016.

[7] A. Emery and A. V. Nenarokomov, Optimal experiment design *Measurement Science and Technology*, vol. 9, no. 6, p. 864, 1998.