Only the Groucho number ensures dynamic similarity during walking

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Abstract: In this paper, we present a method to assess dynamical similarity in bipedal walking based on a dimensionless bipedal spring-mass model (BSMM). We first introduce a new formulation of the BSMM based on the Groucho number. We discuss, afterwards, how to experimentally measure the necessary parameters. The use of the dimensionless bipedal spring-mass model permits to evaluate whether two systems operate dynamically similar, and to judge whether these systems walk in a self-stable region.

Keywords: bipedal spring-mass model, dynamic similarity, bipedal locomotion, Groucho number.

1. INTRODUCTION

Not only Humans are able to walk bipedally. Also many avian species are successful as semi or complete terrestrial animals. Interestingly, the evolutionary constrains imposed by the avian body plan leads to a different geometrical construction of avian legs compared with those in humans. Based on theropod hindlimb configuration, that of birds was highly modified during evolution. The femur was shifted to a more horizontal orientation and relatively shorted. Bones of the distal limb segments have been fused and modified, leading to the tibiotarsus and tarsometatarsus. Overall changes in hindlimbs geometry are understood as adaptations to cursorial locomotion, to leap from the substrate during takeoff, and to absorb the shock of landing [1, 2]. Finally, birds do not walk with the soles of their feet, like humans do, but on their toes (digitigrade locomotion).

Different patterns in limb proportions and orientation may reflect different segment usages and loadings, and thus different local kinematics and dynamics. On the other hand, that fact does not exclude that global locomotion during walking used by humans and birds could not be dynamically similar. We think, this question was not adequately addressed so far.

Specially since popularized by Alexander [3], the Froude number (Fr) $Fr = u^2/gh$ has been used so far to compare bipedal and quadrupedal locomotion. Here, u is the cursorial velocity, h is the leg length, and g is gravity. The Froude number is directly related to the inverted pendulum, as it actually reflects the ratio between inertial and gravitational forces during walking or running. The inverted pendulum template, however, cannot adequately reflect the dynamics of walking.

The simplest model, which is able to reproduce bipedal gait dynamics, i.e. motions of the centre of mass (CoM) and ground reaction forces (GRF), is the bipedal spring-mass model (BSMM) [4]. Although human legs are complex in structure and control, it seems that they can generate a spring like behavior during walking at moderate speeds [6]. Until now, the BSMM was formulated only dimensional. Results were afterwards converted to dimensionless description using the Fr for speed. But, as stated before, Fr may not be an adequate quantity to characterize dynamic similarity of compliant systems. In that case, dynamic similarity can only be ensured, if both systems have the same Groucho number (Gr), $Gr = u\omega_0/g$. here, u is the cursorial velocity, ω_0 is the natural frequency of the system, and g is gravity [5]. In this paper, we present a method to assess dynamic similarity in bipedal walking based on a dimensionless BSMM. We first introduce a new formulation of the BSMM based on the Gr. We discuss, afterwards, how to measure experimentally the necessary parameters. Finally we demonstrate how we can use this formulation to compare the dynamics of two systems and whether these two systems walk in a self-stable region.

2. METHODS

2.1 Dimensionless bipedal spring-mass model

The template for this study is a dimensionless BSSM. This describes the action of the stance leg(s) by a (two) dimensionless linear spring(s) of leg stiffness $\hat{k} = kl_0/mg$. Note that the dimensionless rest length $\hat{l}_o$ is equal to $\hat{k}$. The dimensionless equations of motion restricted to the sagittal plane are:

$$\hat{\ddot{x}} = -\hat{k}\sum_{i=1}^{a}\left(\frac{\hat{x}_{FPi}}{\hat{l}_i} - \frac{\hat{x}_{FPi}}{\hat{l}_0}\right) + \hat{x}\sum_{i=1}^{a}\left(\frac{\hat{l}_0}{\hat{l}_i} - 1\right) \qquad (1)$$

$$\hat{\ddot{y}} = -1 + \hat{y}\sum_{i=1}^{a}\left(\frac{\hat{l}_0}{\hat{l}_i} - 1\right) \qquad (2)$$

where a = 1 for the single (SSPh) and 2 for the double support phase (DSPh). All following quantities are dimensionless: $\hat{\ddot{x}}, \hat{\ddot{y}}, \hat{x}, \hat{y}$ are the accelerations, and positions of the CoM. $\hat{x}_{FPi}$ is the horizontal distance between CoM and foot (feet) position(s), and $\hat{l}_i$ is the leg length of the stance leg(s). A dimensionless model is characterized by a minimal set of parameters. In our case we chose the dimensionless stiffness $\hat{k}$, the angle of attack α_0, the leg compression λ, and the Groucho velocity Gr.

2.2 Simulation

Simulations start when the supporting leg is oriented vertically ($\hat{x} = \hat{x}_{FPi} = 0$), initial height $\hat{y}_0 = \hat{l}_0 - \lambda$, and horizontal velocity equal to the Gr. The model switches to the DSPh when for leg 2 the condition $\hat{y} = \hat{l}_0 \sin\alpha_0$ is met. Then it returns to SSPh when for leg 1 $\hat{l}_1 = \hat{l}_0$. We exploit the convergence to fixed points to find steady-state locomotion. We map the space (Gr, $\hat{k}$, α) for different λ. If any of these states lies in the basis of attraction of a fixed point, the model converges to steady-state trajectories. We accept a fix point as stable if the simulation reaches 100 steps.

2.3 Experimental Parameters

In order to compare different systems such as humans or animals, four parameters have to be experimentally determined: $\hat{k}$, α_0, Gr, and λ. As leg length in our model is symmetric related to touch down (TD) and take off (TO) events, some assumptions have to be made, in order to determine k (for both humans and birds leg length is longer at TO compared with TD). So we compute k as $k = GRF_{midstance}/\Delta l$, where $\Delta l = ((l_{leg(TD)} + l_{leg(TO)}) / 2) - l_{leg(midstance)}$. α_0 represents leg orientation with respect to the ground at TD. Gr is the cursorial velocity, and λ can be obtained as $\lambda = GRF_{midstance}/BW$.

3. RESULTS, DISCUSSION & FURTHER WORK

Depending on the chosen parameters, the model converges or not to steady-state locomotion. As we fix λ, we can also investigate whether the fixed points are shifted to lower or higher values of λ compared to its starting value (basin of attraction). As an example Fig. 1 displays fixed points for simulations started at a λ = 0.9. Six stable walking sub-domains are revealed.

At lower Gr velocities, the number of peaks in the vertical GRF increases up to seven. In Fig. 1, the fix points are presented divided into three clusters, depending on the final λ value (black 0.8 ≤ λ ≤ 0.95, magenta λ < 0.8 and blue λ > 0.95). As expected, stable solutions do not exist for λ > 1 in the M-shape sub-domain. Furthermore, only a discrete set of start parameters result in stable operation at λ values close to 0.9. Our results show that for an even number of peaks, λ should be lower as 1, while for uneven number of peaks, it should be higher than 1. This can be explained mostly by means of the symmetry of the GRF. On the other hand, it seems that λ never reaches the value of 2. Using data from the literature [6] and the method explained in section 2.3 we had been able to locate a point which represents human walking at 1.15 m/s. Normal values of λ at those speeds seem to oscillate about ~ 0.85 [6].

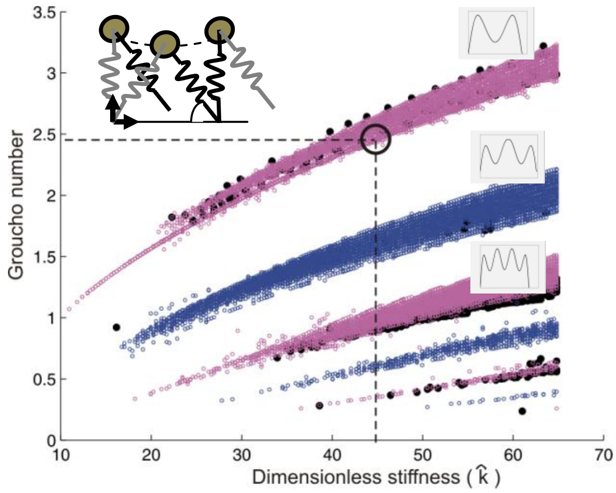

Fig. 1 Dimensionless stable walking sub-domains using leg compression λ = 0.9 as start parameter.

The point computed is located inside the stable region between fix points of λ values close to 0.9 and below 0.8 (Fig.1). Lower values of λ increase the width of the M-shape sub-domain, and diminish that of the multi-peak sub-domains. Lower values of λ increase also the vertical excursion of the CoM, which may lead to an increase in the cost of transport. Therefore, it seems that humans use rather self-stable walking regions, which minimize energy consumption (system parameters optimized for "endurance walking").

Also the analysis of the influences of body-size (child, adult), or leg-stiffness (young, old) on walking dynamics must be based on an adequate dimensionless formulation.

We are now performing, in cooperation with the Inst. of Systematic and Evolutionary Zoology of the University of Jena, kinematic and dynamic studies on small birds like quails using high-speed x-ray motion analysis and custom designed force plates. Results of these experiments will be examined with respect to dynamic similarities or differences.

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