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A Bidirectional Weak Coupling Approach To Rhythmic Movement

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Abstract

This paper describes the properties of a set of simple neural network oscillators suited to two robotic tasks. One robotic task is "wall-bouncing," in which the robot repeats the process of hitting balls that rebound from the wall. Another robotic task is "passing a ball," in which two robots repeat the process of passing balls to each other. The motions of the robot (paddle) are controlled by a set of neural oscillators consisting of four weakly coupled Bonhöffer-van der Pol (BVP) oscillators. We demonstrate that rhythmic movement of the paddle emerges as a stable limit cycle generated by the global entrainment between the paddle, the neural system, and the environment, including balls.

1. Introduction

This paper describes the properties of a set of simple neural network oscillators suited to two robotic perceptual-motor tasks incorporating rhythmic movement. The first robotic task is "wall-bouncing," in which a robot repeats striking and returning two balls rebounding from a wall. We assume that touch sensors are attached to a robot's paddle and the wall. Only the timing of a ball contacting the paddle and the wall is input to the robot. The timing of the paddle movement is adjusted by using a proposed robotic oscillator, and the robot can repeatedly strike and return each ball. Furthermore, by using two balls, we can observe rhythm bifurcations of the robotic oscillator. The rhythm bifurcations are generated by the difference between the phases of balls. We will associate this result with the change of gait in locomotion.

The second robotic task is "passing a ball," in which two robots repeat passing two balls between each other. The identical rhythm oscillator controls each robot. We assume that a touch sensor is attached to each robot's paddle, and the timing of the ball contacting the paddle is detectable. An important point of this task is to transfer the rhythm information through the environment between the robots. Coordinated motion of robots is developed in this task. Here, each robot can be regarded as one large oscillating object. The robotic oscillating objects mutually synchronize through the balls, performing the task successfully. Nature has many examples in which synergy of some oscillators generates oscillation in groups, such as the Southeast Asian synchronously flashing fireflies [1]. This self-organized pattern formation is a collective phenomenon and results from the interaction of a large number of subsystems. We will relate this robotic task to such a phenomenon.

The two tasks involve a type of interaction between a robot (paddle) and the environment (balls) called bidirectional coupling [2]. These tasks can be performed using the same conceptual architecture without modeling of the environment. Furthermore, touch sensors are used in these tasks, not visual sensors. Our approach does not require continuously monitoring the environment, and can be interpreted as a weak coupling [2]. The mirror-law approach by Koditschek et al. [3] can be interpreted as a strong coupling that forces a robot to move the paddle according to the state of the ball at every moment [2]. In contrast, our approach exploits the entrainment between the paddle, the neural system, and the balls, on the condition that only the time a ball contacts the paddle is available.

For these robotic tasks, we propose a system of neural oscillators consisting of four weakly coupled BVP oscillators. A BVP oscillator is a simple neural model expressing the response of the Hodgkin-Huxley (H-H) model, qualitatively equal [4]. We call this a bottom-up fork connected (BFC) robotic rhythm oscillator and note there are multiple BVP oscillators with bottom-up fork connection through the rhythm kernel. The BFC robotic rhythm oscillator inputs touch sensor information and outputs a paddle drive timing. Williamson has proposed a simple neural oscillator coupled to a real robot arm and has demonstrated that the system is capable of coordinated motion without global synchronization or control due to the oscillator's entrainment property [5]. His study is regarded as an application of Taga's idea for human locomotion [6], that is, "global entrainment," to a robot arm. Although his approach is similar to ours in that the perceptual-motor coupling is bidirectional, the model and structure of his oscillator differs from ours, and the perceptual-motor coupling is strong. Williamson's oscillator output waveform is used to command the arm joints. This has limitations in that the output waveform has a constant shape and lacks flexible movement. In contrast, our approach employs the oscillator output as a starting cue of the paddle movement.

Shannon studied the bounce-juggling task in which a robot repeats bouncing a ball on a floor and capturing it with the paddle [7]. He has shown that a certain fixed pattern of the paddle movement can achieve this task if appropriate parameters, such as the drive frequency and the amplitude of paddle, and the height of equipment, are chosen. This task is similar to the wall-bouncing task discussed in this paper. His approach is categorized as unidirectional coupling because the ball has no effect on the paddle's motion, while our approach, bidirectional coupling, involves coupling from the ball to the paddle.

Schaal et al. investigated the task of one-handed ball bouncing with a paddle, postulating that humans exploit the dynamic stability of this task [2]. They indicated that unidirectional coupling is dominant in the human ball-bouncing task. Beek et al. studied the timing selection of rhythmic catching in human behavior [8]. They revealed a constant time interval between the zenith of the ball's trajectory and the initiation of the catch from an analysis of the hand's trajectory. They subsequently hypothesized that humans may use time-to-contact information about the ball's zenith to time the catch appropriately. In reality, although information from the ball's trajectory is critical for a successful task, continuous visual tracking along the entire trajectory is not necessary. In the juggling instruction, "Look at the highest point," and "Throw the next ball when the previous one reaches the top" are common teaching [9]. Humans control the timing of throwing and catching without the ball's entire flight information. From this point, their hypothesis is reasonable. However, it is reported that some professional jugglers can juggle balls blindfolded. Expert jugglers depend more on the sensation achieved between the hand and ball contact, whereas novice jugglers rely predominantly on their eyes [9]. This shows that tactile information about the ball contact can substitute for visual information. Schaal et al. also studied the one-handed bouncing ball task, excluding various perceptual information of human behavior [10]. They concluded that kinetic information about the impact is more necessary than visual information, although the latter gives information about the continuous kinetic trajectory of the ball.

Kotosaka et al. developed an imitation robot by exploiting the entrainment properties of nonlinear oscillators, although their approach is unidirectional coupling[11].

In contrast with these previous studies, our approach is close to human juggling, because all sensors are only used to obtain timing information in rhythmic movement and our tasks are categorized as bidirectional coupling. In particular, we consider the passing a ball task to be equivalent to professional blindfold juggling (except for the dynamic effects on the balls, such as gravity) because only the paddle's touch sensor is used to obtain perceptual information.

The proposed BFC robotic rhythm oscillator can be interpreted as being inspired by the mechanism of the reflection generated by perception and the Central Pattern Generator (CPG). We know that the CPG can modify the reflection pattern by using sensor inputs as well as higher-level brain commands. The reflection here can be regarded as a solution of inverse kinematics. The BFC robotic rhythm oscillator autonomously acquires this solution due to the oscillator's entrainment property. This paper focuses on the following two features of nonlinear oscillators. (1) The entrainment and input/output properties of oscillators enable robots to perform a variety of tasks with the same conceptual architecture. (2) The motion emerging from the local interaction of the oscillators and entrainment (weak coupling) leads to stable performance of the whole system.

Section 2. of this paper describes the architecture for rhythmic movement generation and includes the details of the BFC robotic rhythm oscillator. The following sections present robotic task simulations using the BFC robotic rhythm oscillator, the wall-bouncing task (section 3.) and the passing a ball task (section 4.). Section 5. concludes the paper with a discussion.

2. Architecture for Rhythmic Movement Generation

2.1. Robotic brain system

2.1.1. Higher-levels in robotic brain system

Our robotic system is designed based on the hypothesis of the multi level control system of movements (Fig.1). In this system, the higher-levels determine the general characteristics in the task, such as start, stop, faster, slower and so on. These commands correspond to the value of a single parameter z in the following section 2.1.3..

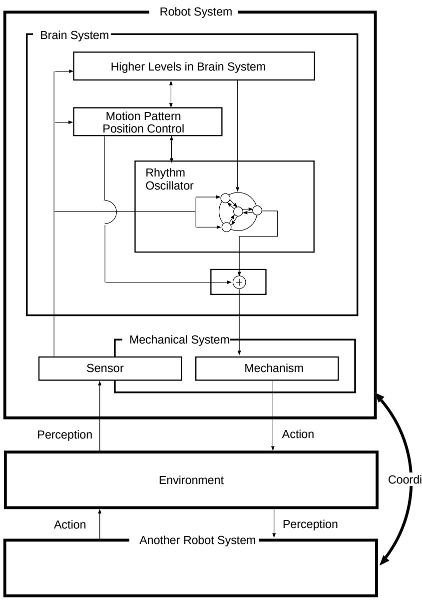

Figure 1: Perceptual-motor system for rhythmic movement

2.1.2. *Motion pattern and position control*

Our system considers the pattern and tempo (timing) of motion separately. Here, the motion pattern is fixed, that is, a trapezoid pattern, on the time vs. velocity map, as shown on the left side of Fig.4.

The rhythm oscillator shown in the next section adapts to the relative change of the environment. However, a robot needs to control the position of the paddle specified in an external coordinate frame fixed to the ground. We adjust the timing calculated by the rhythm oscillator so as to keep an ideal hitting point. By shifting the timing, we control (the timing of) the motion discretely. This is the “active” control in an inertial coordinate frame.

2.1.3. *Rhythm oscillator*

The rhythm oscillator must adapt to the environment and conform to the higher-level commands in the robotic brain. To solve this problem, we utilize the property of a neural oscillator (nonlinear oscillator) with a tonic input. Our proposed rhythm oscillator can adapt to the external condition “passively” and can be also ruled by higher-level commands, for example START, STOP, CONTINUE.

Rhythm generation:
In this paper, we propose a system of neural oscillators consisting of four weakly coupled Bonhöffer-van

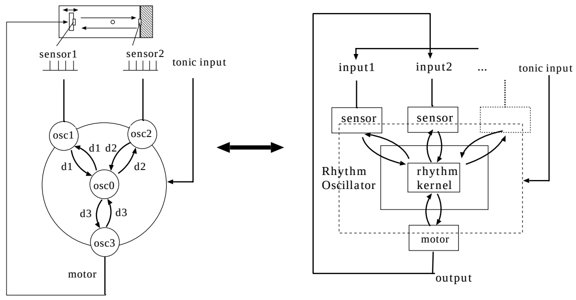

Figure 2: General idea of rhythm oscillator for some robotic tasks

der Pol (BVP) oscillators named the bottom-up fork connected (BFC) robotic rhythm oscillator as a robot’s rhythm oscillator (Fig.2). Although a BVP oscillator is a simple neural model with two variables, this model can qualitatively express the response of actual neurons very well [4]. Moreover, a simple BVP oscillator and its some coupled ones have been analyzed in detail from the viewpoint of rhythm bifurcations. The proposed robot’s rhythm oscillator (BFC robotic rhythm oscillator) can be divided into three units: a sensor unit (osc1 and osc2); a rhythm unit (osc0); and a motor unit (osc3). Each sensor oscillator (osc1 and osc2) receiving sensor inputs is combined with a rhythm core oscillator (osc0), and information is transmitted synchronously. Similarly, information is output from the motor oscillator (osc3) as a result of mutual combination with a rhythm core oscillator. The rhythm core, which is the central unit of the BFC robotic rhythm oscillator, is not directly connected with the system’s inputs and outputs. Sensor fusion and selection is easily adjusted by tuning coupling coefficients (d_1 and d_2). If the rhythm oscillator has to handle more sensors, we can employ the same architecture as Fig.2 by providing additional sensor oscillators.

The model equation of each oscillator constituting the BFC robotic rhythm oscillator is given below:
Rhythm Core Oscillator (osc0):

$$\left\{ \begin{array}{lcl} \frac{dx_0}{dt} & = & c\{y_0 + x_0 - \frac{1}{3}{x_0}^3 + d_1(x_1 - x_0) \\ & & +d_2(x_2 - x_0) + d_3(x_3 - x_0) + z\} \\ \frac{dy_0}{dt} & = & -\frac{1}{c}(x_0 + by_0 - a) \end{array} \right. \tag{1}$$

Sensor Oscillator 1 (osc1):

$$\left\{ \begin{array}{lcl} \frac{dx_1}{dt} & = & c\{y_1 + x_1 - \frac{1}{3}{x_1}^3 + d_1(x_0 - x_1) \\ & & +z\} \\ \frac{dy_1}{dt} & = & -\frac{1}{c}(x_1 + by_1 - a) \end{array} \right. \tag{2}$$

Sensor Oscillator 2 (osc2):

$$\left\{ \begin{array}{lcl} \frac{dx_2}{dt} & = & c\{y_2 + x_2 - \frac{1}{3}{x_2}^3 + d_2(x_0 - x_2) \\ & & +z\} \\ \frac{dy_2}{dt} & = & -\frac{1}{c}(x_2 + by_2 - a) \end{array} \right. \tag{3}$$

Motor Oscillator (osc3):

$$\begin{cases} \frac{dx_3}{dt} &= c\{y_3 + x_3 - \frac{1}{3}{x_3}^3 + d_3(x_0 - x_3) \\ & \quad +z\} \\ \frac{dy_3}{dt} &= -\frac{1}{c}(x_3 + by_3 - a) \end{cases} \quad (4)$$

System parameters are fixed as a = 0.7, b = 0.8, and c = 3.0 in the following robotic tasks given in Sections 3. and 4.. (We discuss the coupling coefficient (d_1, d_2 and d_3) in the section 3.2..) The parameter z is a tonic input, and is controlled by the higher-levels of the brain system. If z = 0.0, the BFC robotic rhythm oscillator does not fire, even if the stimulation is added to a robot (STOP). When the parameter z is decreased (we fixed z = - 0.2 in the following tasks), the BFC robotic rhythm oscillator works according to the sensor input (START, CONTINUE). In this BVP oscillator model, note that a negative input corresponds to an excitation stimulus. We assume that a sensor pulse instantaneously shifts the membrane potential x to $x-h$, when sensor signal inputs to a sensor oscillator (osc1 or osc2) [4]. The parameter h is also fixed as h = 1.0.

In addition, referring to bifurcation diagrams of a (not connected) BVP oscillator stimulated by periodic pulse trains [4], we adopted different time scales between the rhythm oscillator and the real world, that is, 10:1.

Compensation of phase difference:
In forced synchronization, the phases of an external signal and an oscillator cannot be locked in general, although their frequencies can be synchronized by frequency entrainment. This paper coarsely matches phases by shifting time through task simulations executed before actually performing the task. This process is equivalent to an imaginary rehearsal, visualizing the rhythm of movement to be performed in his mind, so we call this period the "adaptation stage."

A robot actually performs a task after the adaptation stage, and we call this period the "execution stage." The perturbation of the rhythm roughly tuned in the adaptation stage is compensated by the frequency entrainment of a rhythm oscillator. Thus, the time shift for phase compensation is realized by the following two effects: (1) coarse phase shift - the phase shift in the adaptation stage based on a coarse model of the phase difference between an external signal and an oscillator, and (2) fine phase shift - the phase shift in the execution stage achieved by the frequency entrainment of a rhythm oscillator.

In fact, the strike timing is shifted by both the feedback (section 2.1.2.) and the frequency entrainment in the execution stage. This operation exploits both the discrete control and rhythmic control [11]. The rhythm oscillator changes time interval dynamically, while the position feedback adjusts the timing statically. The task is performed successfully by the synergy of these effects.

2.2. Mechanical system

We assume that touch sensors are attached to a robot's paddle and wall (or partner's paddle). Only the timing of a ball contacting the paddle and the wall (or partner's paddle) is input to a robot in the execution stage. Moreover, a coefficient of restitution of the paddle is assumed to change at random in a certain range as perturbation. This will make a change in the ball speed, which leads to a change in the timing of sensor inputs.

3. Wall-Bouncing Task with Two Balls

3.1. System configuration

We simulate the wall-bouncing task in which a robot repeats striking and returning a ball that rebounds from a wall as depicted in Fig.3. The ball is always rolling on the guide set up on a horizontal plane. This task is equivalent to the one-handed fountain juggling except for the dynamic effects on the balls, such as gravity. Remember the juggling instruction to "look at the highest point." The player looks at the zenith of the balls' trajectory and perceives the timing at which the balls pass through the zenith. The touch sensor on the wall corresponds to the player's sight in this robotic task, while the touch sensor attached to the robot's paddle corresponds to the player's tactility.

In this simulation, the ball speed varies approximately from 480 to 520 [mm/sec] owing to perturbation. The ideal movement, simulated in the adaptation stage assuming that a robot can hit a ball stably at a fixed position with a fixed speed, continues for 50 [sec] preceding the execution stage in which the sensor information is acquired and the rhythm oscillator is actually driven. The peak time of the output of the motor oscillator is given as a command cue to the paddle motor whose motion parameters are predetermined. The model figure of this task is shown in Fig.4.

3.2. Simulation results

In the following simulation (Figs.5 to 7), the coupling coefficient of each oscillator (d_1, d_2, and d_3) is fixed as d_1 = d_3 = 0.067, d_2 = -0.067. Fig.5 compares between two cases: the case in which the paddle is driven at a fixed frequency, and the case in which it is driven using

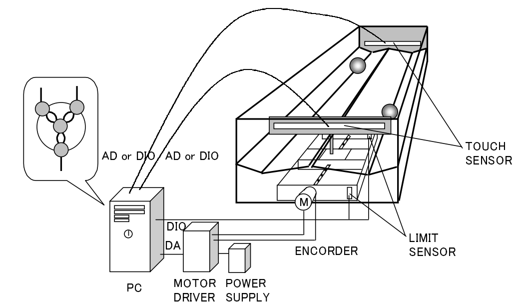

Figure 3: Wall-bouncing task using rhythm oscillator

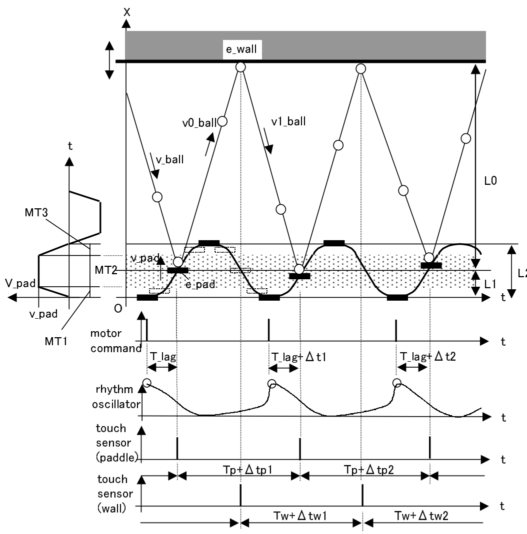

Figure 4: Wall-bouncing task model

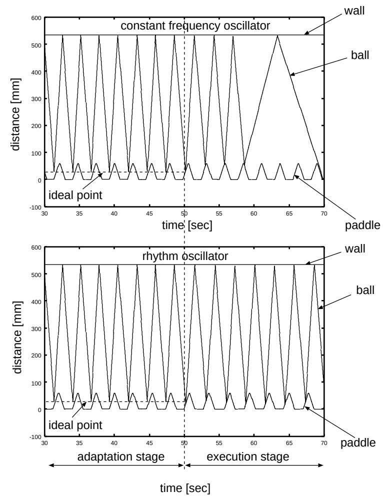

Figure 5: Effect of rhythm oscillator in wall-bouncing task: constant frequency oscillator (up), rhythm oscillator (down)

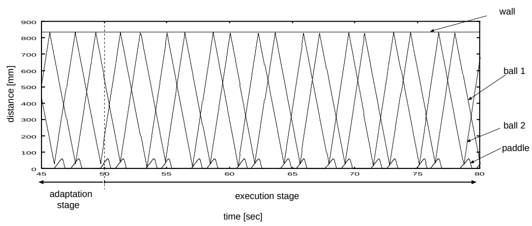

Figure 6: Two balls' trajectories in wall-bouncing task with two balls

the rhythm oscillator. In this example, only one ball is used for simplification. When the paddle is driven at a fixed frequency, the task ends in failure due to the influence of perturbations. However, when the paddle is driven with a rhythm oscillator, it absorbs perturbations and continues hitting the ball. Figure6 illustrates the result of the wall-bouncing task using two balls. Though the rhythm oscillator cannot distinguish one ball from the other, like the task using one ball, the timing of paddle movement is adjusted and the paddle can strike each ball repeatedly. Figure7 shows the output signal of each oscillator in the wall-bouncing task with two balls. Here, while each sensor oscillator (osc1 and 2) is generally anti-phase, the rhythm core oscillator, the sensor oscillator on the paddle, and the motor oscillator are in-phase. Each of these rhythms settles down in a certain fixed limit cycle, and leads to a stable periodic solution of the whole dynamic system including the environment (balls).

Figure8 presents two different stable motion patterns in the wall-bouncing task with two balls. These different patterns result from the structural difference of the rhythm oscillator. One stable pattern results from the rhythm oscillator model in which all oscillators (osc0, osc1, osc2, and osc3) are connected with reciprocal inhibition. We call this model "8000" using the site-swap notation [9], which is one of the popular juggling notations. Site swaps are a compact notation representing the order in which balls are thrown and caught in each cycle of juggling, assuming throws happen on beats that are equally spaced in time. Although details are omitted, we know that there is a close relationship between site swaps and mathematics. Another stable pattern results from the rhythm oscillator model in which one sensor oscillator (osc2) is connected to a rhythm core oscillator (osc0) with reciprocal excitation while other oscillators (osc1, osc3) are connected to a rhythm core oscillator with reciprocal inhibition. We call this model "80800000" using the site-swap notation. In the model "8000," coupling coefficients are described as $d_1, d_2, d_3 > 0$; in the model "80800000," they are described as $d_1, d_3 > 0$, $d_2 < 0$. In the former

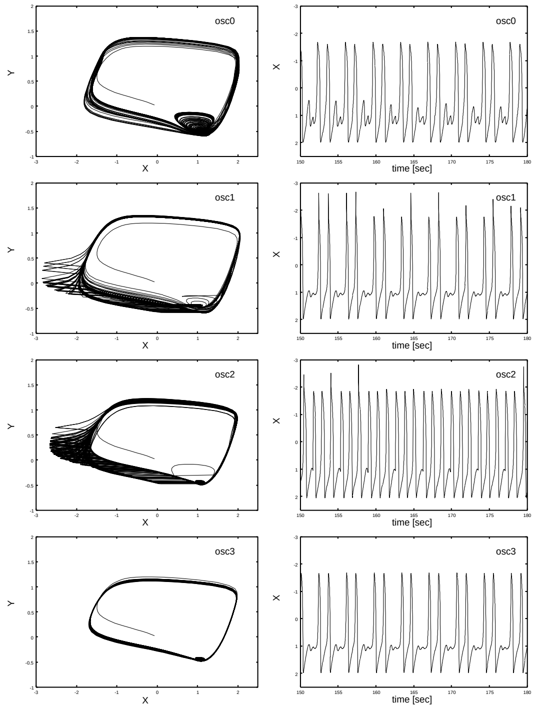

Figure 7: Output signal of each oscillator: phase plane (left), x coordinate (right)

model, we fixed the coupling coefficient parameters as d_1 = d_2 = d_3 = 0.067, while we fixed them as d_1 = d_3 = 0.067, d_2 = - 0.067 in the latter model. Even if a robot starts to perform this task on the same initial condition of balls, the timing of hitting a ball settles down in a different pattern according to the difference of the rhythm oscillator models. When we choose the rhythm oscillator model "8000," each ball is hit at the same interval. In contrast, when we choose the model "80800000," the two balls are hit at different time intervals. If this task is performed with one ball, note that we cannot use the model "8000" as the rhythm oscillator model because the paddle sensor is out of phase with the wall sensor. Both input signals to sensor oscillators must be in-phase for the model "8000."

Moreover, to show how this system adapts to the change of environment, we simulate the moving-wall-bouncing task with two balls. A change in the distance between the robot and wall leads to a change in the timing of hitting. A coefficient of restitution of the paddle also changes at random in a certain range as perturbation. We fixed the coupling coefficient parameters as $d_1 = d_2 = d_3 = 0.067$ in this simulation. Figure9 depicts the simulation result. A robot changes the time interval of hitting according to the change of environment. This effect enables a robot to keep hitting two balls successfully at almost equally space in time. Moreover, the position feedback effectively keeps the ball contact position around an ideal location. In this simulation, we change the tonic input z from z=-0.2 to z=0.0 at 900.0 [sec] as a higher-level brain command of brain system, STOP. The robot stops the paddle, obeying this command.

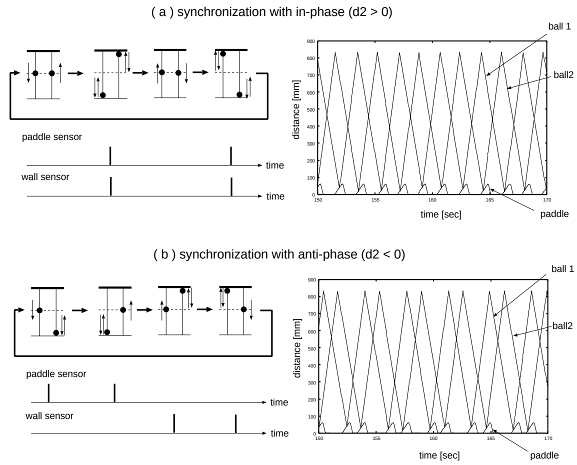

Figure 8: Two stable states in wall-bouncing task with two balls: model "8000"(up), model "80800000" (down)

In addition, we realize two advantages by using the BFC robotic rhythm oscillator. First, in the wall-bouncing task with two or more balls without this oscillator, even if the paddle continues hitting balls at an ideal location, the difference between phases of balls varies due to perturbations, and the paddle occasionally has to hit two or more balls almost simultaneously. However, such a wrong case does not occur when the rhythm oscillator is used because the rhythm oscillator maintains a fixed rhythm pattern. Second, the system parameters of the rhythm oscillator are designed so that a rhythm oscillator has a 1:1 phase-locking region in the broad range of inputs' periods. Thereby, even if the robot misses one ball, its rhythm oscillator synchronizes with another ball's period immediately and the robot can continue hitting another ball successfully (Fig.10).

The output pattern of the rhythm oscillator changes according to the difference between ball phases. The rhythm bifurcation can be seen through this task. In Fig.11, the left side of the graph shows the type 1:1 - the membrane model fires periodically every time it is stimulated, and the right side of the graph shows the type 2:1 - the membrane model fires in units of two times. This result is very interesting from a motion control viewpoint, but this bifurcation may sometimes

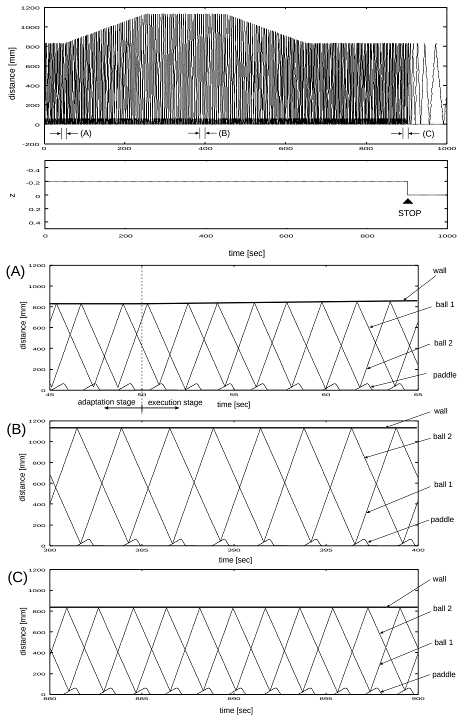

Figure 9: Two balls' trajectories and their enlargements in moving-wall-bouncing task

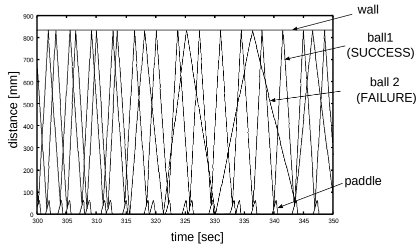

Figure 10: Robot keeps hitting one ball while it fails in hitting another ball

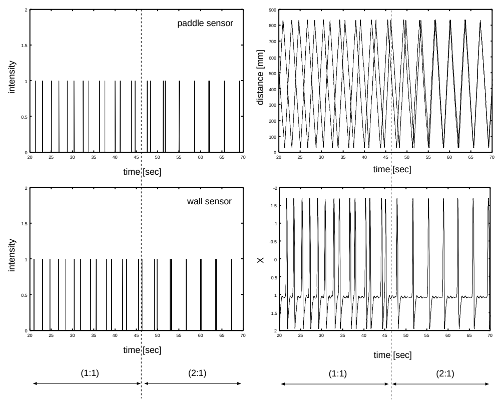

Figure 11: Bifurcation of rhythm : paddle sensor (left-up), wall sensor (left-down), ball's trajectory (right-up), x coordinate of motor oscillator (right-down)

cause the task to end in failure.

4. Two Robots Passing Two Balls

4.1. System configuration

The passing a ball task with two balls by two robots facing each other is simulated (Fig.12). It is assumed that the touch sensor is attached to each robot's paddle and the contact timing of a ball to the paddle is detectable, while the rhythm oscillator can not distinguish one ball from the other. Furthermore, the distance between two robots changes and a coefficient of restitution of the paddle also changes at random in a certain range as perturbation. The identical rhythm oscillator controls each robot. Only the contact timing of a ball to the robot's paddle and its partner's one is given to each robot as an input. This is equivalent to the situation in which the robots pass balls while informing each other of the impact. This task is similar to the two-ball blind passing juggling with a shout except for the dynamic effects on the balls. The touch sensors attached to the robots' paddle correspond to players' tactility. The touch sensor attached to the one robot's paddle also corresponds to the another player's hearing, because players inform the timing of ball impact with a shout to each other. The connection of rhythm oscillators in passing a ball task is shown in Fig.13.

4.2. Simulation results

In this simulation, we fixed the coupling coefficient parameters as $d_1 = d_2 = d_3 = 0.067$. This is the same as the model "8000" in section 3.2.. In this model, all oscillators are synchronized mutually in-phase, and each oscillator settles down in a certain fixed limit cycle, leading to a stable periodic movement of the whole system. Figure14 shows output signals of each oscillator constituting the rhythm oscillator of one robot in the task of two robots passing two balls. Here, each robot can be regarded as one large oscillating object consisting of a set of oscillators. These oscillating objects mutually synchronize through balls and perform the task successfully by adapting to the environment. Each robot's rhythm core oscillator synchronizes in-phase. As a result, the robots can continue hitting balls repeatedly even if one of them is moving (Fig.15).

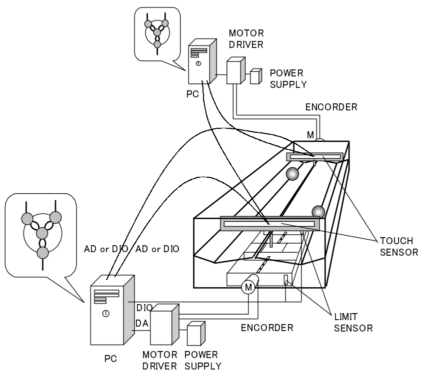

Figure 12: Passing a ball task using rhythm oscillator

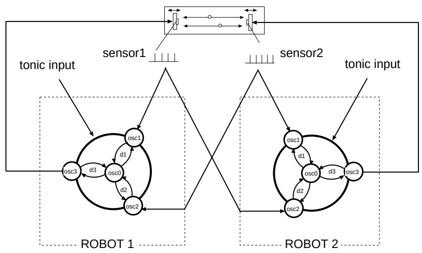

Figure 13: Connection of rhythm oscillators in passing a ball task

5. Conclusion

In this paper, we proposed a system of neural oscillators called the BFC robotic rhythm oscillator that enables robots to perform two perceptual-motor tasks. Using the BFC robotic rhythm oscillator, we confirmed that a robot autonomously generates stable rhythmic movement without any global synchronization or control, due to the local interaction of the oscillators and their entrainment properties. Our approach, bidirectional weak coupling, also suggest the importance of kinetic information (timing information) about the impact in rhythmic movement. In future work, we will show that two-hand coordination also emerges as a stable limit cycle generated by the global entrainment between the limbs, the neural system, and the environment. Using a real robot should reveal the effectiveness of this approach.

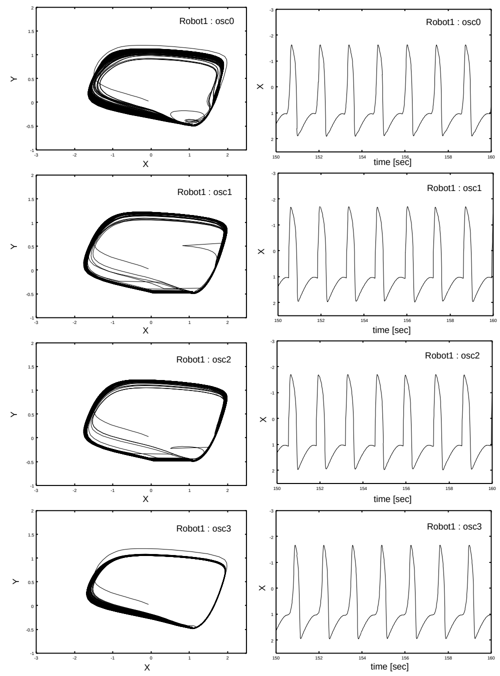

Figure 14: Output signal of each oscillator (robot1): phase plane (left), x coordinate (right)

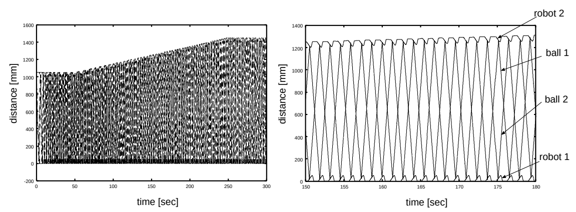

Figure 15: Two balls' trajectories and their enlargements in passing a ball task with two balls by moving robots

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