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An analytical study of the cost of transport for legged locomotion

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Abstract

It has been known that locomotor patterns of many legged animals show common characteristics, which suggests that there exists a basic strategy for legged locomotion. In this study we examine the minimization of the cost of transport as a candidate of the strategy. For this purpose we derive an equation to estimate the cost of transport during legged locomotion and show that the optimal locomotor pattern minimizing the cost well represents many characteristics of the observed pattern in legged animals. The results suggest that the locomotor pattern of legged animals are well optimized on the energetic cost and the existence of specific gait patterns and the phase transition between them are also results of the optimization.

1. Introduction

Locomotor patterns of many legged animals, such as a horse, a cat, a cockroach, and a crab, show common characteristics as follows in spite of large difference of body size, body structure, and number of legs as suggested by Full and Tu (1990). (1) A stride period decreases with locomotion velocity and reaches an almost constant value but a swing duration takes an almost constant value, which results in the decrease of duty ratio, the ratio of the stance duration, the duration a leg is in contact with the ground, to a stride period with locomotion velocity [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The decrease of the duty ratio also means the decrease of the average number of stance legs. (2) The stride length, the forward distance the body moves in a stride period, and the stance length, the forward distance in a period in which a foot applies force to the ground to support the body, slightly increases with locomotion velocity or keeps almost constant value [2, 3, 8, 11, 12, 13], and begins to increase after the stride period reaches an almost constant value [8]. (3) The gait pattern tends to show non-graded change and stays at some specific patterns such as walking and trotting [14, 15]. In quadrupods such as a horse three typical gait patterns are observed, and in six legged insects five typical patterns are reported [9]. The transition between gaits occurs in phase transition manner in four legged mammals, however, the existence of graded transition has been also suggested in insects [16, 17]. (4) The cost of transport, the energetic cost to move a unit mass a unit distance, decreases with velocity and becomes an almost constant value [1, 4, 12, 18, 19, 20, 21, 22, 23].

The existence of these common characteristics indicates that there exists a basic strategy for the choice of a gait pattern in many legged animals. As a candidate of the strategy the minimization of the cost of transport was suggested by Hoyt and Taylor (1981) by an experiment measuring the oxygen consumption during horse locomotion. Recent studies by Nishii (1998,2000) showed that many characteristics of the optimal locomotor pattern minimizing the cost of transport estimated for a dynamical model of insects are coincident with observed locomotor patterns. In the latter study the optimal locomotor pattern was computed by using inverse dynamics method, however, no explicit equation to estimate the energetic cost during locomotion was not given, which makes difficult to understand the detailed account of the cost.

In this study we will derive an equation to estimate the cost of transport and show that the criterion of the minimization of the cost given by the equation explains many characteristics of legged locomotions.

2. Estimation of the energetic cost of transport

As parameters to define a gait pattern we consider duty ratio β, a stride period T, and stance length S. We assume that the values of these parameters are the same for all legs respectively, the stance length is much shorter than a leg length, and the body moves with almost constant velocity V. These parameters satisfy the following relation.

$$V = S/\beta T. \tag{1}$$

Therefore we must determine two parameters to specify a gait pattern. The relative velocity of i-th foot $\dot{x}_i$ to

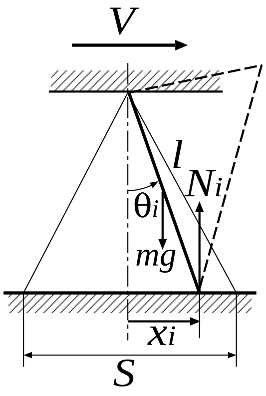

Figure 1: A simple leg model. Although a leg of most legged animals is a multiple link system with multiple joints (e.g., see dashed line), we regard it as a simple one-link system in the estimation of the mechanical work.

the body during stance phase is given by $\dot{x} = -V$ and we assume that the relative foot velocity during swing phase takes sinusoidal form, i.e.,

$$\begin{aligned} \dot{x}_i(t) =& a + b\cos\frac{2\pi}{(1-\beta)T}t,\ (0 \le t < (1-\beta)T) \\ a =& \frac{\beta V}{1-\beta}, \quad b = -\frac{V}{1-\beta}, \end{aligned} \tag{2}$$

where $t = 0$ is the time when the swing phase starts. The position and the acceleration of a foot required for the analysis in latter sections are given by the integration and the differentiation of the above equation.

2.1. Mechanical work to move a leg

Although a leg of legged animals is composed of multiple links with multiple joints, we estimate the required mechanical work to move a leg in a stride period by regarding a leg as a simple one-link system with one joint (Fig:1) and by ignoring the effect of the gravity on a leg. We ignore the cost by negative mechanical work because muscles consume positive energy even for negative work but the energy loss is much smaller than that for positive work with the same absolute value [26]. The mechanical work to move i-th leg W_i is given by

$$W_i = \int_T f(\tau_i \dot{\theta}_i)dt, \tag{3}$$

where τ_i and $\dot{\theta}_i \simeq \dot{x}(t)/l$ are the joint torque and the angular velocity of i-th leg, respectively, l is the length of a leg, and $f(x) = x$ for $x \ge 0$ and $f(x) = 0$ for $x < 0$.

By putting the inertia moment of a leg around its joint as I, the joint torque during swing phase τ_i^{sw} is given by

$$\tau_i^{sw}(t) = I\ddot{\theta}_i(t) \simeq I\ddot{x}_i(t)/l. \tag{4}$$

Equations (2) and (4) give the mechanical work during swing phase W_i^{sw} as

$$W_i^{sw} = \int_{T_{sw}} f(\tau_i^{sw}\dot{\theta}_i)dt \simeq I\left(\frac{V}{l}\right)^2 \frac{1+\beta^2}{(1-\beta)^2}, \tag{5}$$

where $T_{sw} = (1-\beta)T$ is the swing duration. This term corresponds to the work supplying the kinetic energies for the maximum angular velocities during stance phase $\frac{V}{l}$ and swing phase $\frac{1+\beta}{1-\beta}\frac{V}{l}$. The mechanical power, the work in a unit time, given by the above equation divided by a stride period takes the same form as the power derived by Minetti (1998) except the constant coefficient.

During stance phase the angular velocity of a leg is $\dot{\theta} \simeq -V/l$ and the joint torque τ_i^{st} takes

$$\tau_i^{st}(t) = -N_i(t)x_i(t), \tag{6}$$

where $N_i(t)$ is the ground reaction force for i-th leg. Therefore the mechanical work during stance phase W_i^{st} is given by

$$W_i^{st} \simeq \int_{T_{st}} f(N_i(t)x_i(t)) \cdot \frac{V}{l}dt, \tag{7}$$

where $T_{st} = \beta T$ is the duration of the stance phase. From eq. (5) and (7) we obtain the total mechanical work:

$$\begin{aligned} W_i &= W_i^{sw} + W_i^{st} \\ &\simeq I\left(\frac{V}{l}\right)^2 \frac{1+\beta^2}{(1-\beta)^2} + \int_{T_{st}} f(N_i(t)x_i(t)) \cdot \frac{V}{l}dt. \end{aligned} \tag{8}$$

2.2. Heat energy loss due to force generation

Muscles consume energy in force generation even if no mechanical work is done such as in isometric contractions, and the consumed energy is lost as heat energy. During locomotion muscles consume energy by such heat emission in the force generation to support the body and to produce mechanical works to move legs. Because the relation between muscle force and heat energy loss during locomotion has not been known in physiological experiments, we assume that the cost H is proportional to the k-th power of produced joint torque τ, i.e., $H(\tau) \propto |\tau|^k$.

Under this assumption we estimate the heat energy loss for i-th leg. During swing phase the heat energy loss H_i^{sw} is due to torque to swing a leg (eq. (4)) and given by

$$H_i^{sw} = \gamma \int_{T_{sw}} |\tau_i^{sw}(t)|^k dt. \tag{9}$$

During stance phase the heat energy loss H_i^{st} takes

$$\begin{aligned} H_i^{st} &= \gamma \int_{T_{st}} (|\tau_i^{st}(t)|^k + |\alpha N_i(t)|^k) dt \\ &= \gamma \int_{T_{st}} |N_i(t)|^k (|x(t))|^k + \alpha^k) dt, \end{aligned} \tag{10}$$

where α and γ are constant values. The first term is the cost due to rotational torque generation to move a leg against the ground reaction force. Legs of most insects radially spread from their body, therefore, steady joint torque is required to maintain such posture. In most mammals steady joint torque is also required to maintain a bended leg posture under their body. The second term represents the heat energy loss due to such joint torque.

When the body stands still with n_{st} stance legs and the body weight is equally distributed for all stance legs, i.e., the ground reaction force for each stance leg is $N_i = W/n_{st}$, the total heat energy loss for all legs is proportional to $n_{st}(W/n_{st})^k$. In the case of $k > 1$ distributing the body weight in many legs suppresses the heat energy loss to support the body, because $n_{st}(W/n_{st})^k < m_{st}(W/m_{st})^k$ holds for $n_{st} > m_{st}$.

From eq. (9) and (10) we obtain the total heat energy loss:

$$\begin{aligned} H_i &= H_i^{sw} + H_i^{st} \\ &= \gamma \left\{ \int_{T_{sw}} |\tau_i^{sw}(t)|^k dt + \int_{T_{st}} \left\{ |N_i(t)|^k (\alpha^k + |x(t)|^k) \right\} dt \right\}. \end{aligned} \tag{11}$$

2.3. Cost of transport

Total energetic cost for i-th leg during a stride period E_i is given by the sum of eq. (8) and (11). Therefore, the cost of transport e, the energetic cost to move a unit weight a unit distance, is given by

$$e = \frac{\sum_{i=1}^n E_i}{WVT}, \tag{12}$$

where n is the number of legs and W is the body weight.

Let us try the further analysis of eq. (12) under the assumption: the ground reaction force is given by the body weight W divided by the average number of stance legs $n\beta$, i.e., $N_i = W/n\beta$. When heat energy loss is proportional to the square of the generated torque, i.e., $k = 2$, the cost of transport given by eq. (12) with (8) and (11) takes the following form.

$$\begin{aligned} e(V,\beta,S) &= e_w^{sw} + e_w^{st} + e_h^{sw} + e_h^{st}, \\ e_w^{sw} &\equiv \frac{\sum_{i=1}^n W_i^{sw}}{WVT} = \frac{n}{WVT} \cdot \frac{I}{l^2} V^2 \frac{1+\beta^2}{(1-\beta)^2} \\ &= \frac{nI}{l^2 W} \frac{\beta V^2}{S} \frac{1+\beta^2}{(1-\beta)^2}, \\ e_w^{st} &\equiv \frac{\sum_{i=1}^n W_i^{st}}{WVT} = \frac{n}{WVT} \cdot \frac{W}{8l} \frac{S^2}{n\beta} \\ &= \frac{1}{8l} S, \\ e_h^{sw} &\equiv \frac{\sum_{i=1}^n H_i^{sw}}{WVT} = \frac{n}{WVT} \cdot \gamma \frac{2\pi^2 I^2}{l^2} \frac{\beta V^3}{(1-\beta)^3 S} \\ &= \gamma \frac{2n\pi^2 I^2}{l^2 W} \frac{V^3 \beta^2}{(1-\beta)^3 S^2}, \\ e_h^{st} &\equiv \frac{\sum_{i=1}^n H_i^{st}}{WVT} = \frac{n}{WVT} \cdot \gamma \left(\frac{W}{n} \right)^2 \frac{S}{\beta^2 V} (\alpha^2 + \frac{S^2}{12}) \\ &= \gamma \frac{W}{n} \frac{1}{\beta V} (\alpha^2 + \frac{S^2}{12}), \end{aligned} \tag{13}$$

where e_w^{st} and e_w^{sw} are the costs of transport due to mechanical work during stance phase and swing phase, respectively, and e_h^{sw} and e_h^{st} are the costs due to heat energy loss during swing phase and stance phase, respectively.

The equation (13) suggests that larger duty ratio β suppresses the heat energy loss during stance phase because the force to support the body weight is distributed in many legs, but causes larger costs during swing phase, e_h^{sw} and e_w^{sw}, to move a leg in a shorter duration of swing phase. During swing phase stance length S does not affect the mechanical work and larger stance length gives smaller heat energy loss. Because larger stance length gives smaller step number to move a unit distance, it suppresses the cost to move a unit distance during swing phase, e_w^{sw} and e_h^{sw}. However, larger stance length also results in larger energy loss during stance phase, e_w^{st} and e_h^{st}, because of the increase of required rotational torque against the body weight especially when the leaning of a leg is large. Therefore, the optimal stance length S and the optimal duty ratio β are determined by the balance between the costs due to the torque against the body weight during stance phase and the torque to move a leg during swing phase.

In lower velocities the heat energy loss during stance phase, e_h^{st}, is dominant in the total energy cost in order to support the body, therefore, larger duty ratio β and smaller stance length S would be expected to suppress the cost. On the other hand, in higher velocities the cost during swing phase, e_w^{sw} and e_h^{sw}, is dominant in order to swing legs as suggested by Delcomyn and Usherwood (1973), therefore, smaller duty ratio β and larger stance length S would be expected so as to suppress the cost. From these considerations it is expected that as locomotion velocity increases duty ratio decreases, in other words the number of stance legs decreases, and stance length increases. The increase in stance length, however, might be suppressed by the mechanical work during stance phase e_w^{st}.

When the heat energy loss is assumed to be proportional to the generated force, i.e., $k = 1$, the cost of transport due to heat emission takes the form

$$\begin{aligned} e_h^{st} &= \gamma \frac{1}{V}\left(\alpha + \frac{S}{4}\right) \\ e_h^{sw} &= \gamma \frac{4nI}{lW}\frac{\beta V}{S(1-\beta)} \end{aligned} \tag{14}$$

In this case the total cost of transport monotonously increases with the duty ratio β for all velocities, therefore, the smallest duty ratio always gives the minimum cost, which cannot explain the observed gait transition in legged animals.

In more general case in which the heat energy loss is proportional to the k-th power of generated forces the cost of transport during stance phase is estimated by

$$\begin{aligned} e_h^{st} &= \gamma \frac{n}{WVT}\int_0^{\beta T}\left(\frac{W}{n\beta}\right)^k (\alpha^k + |x(t)|^k)dt \\ &= \gamma \left(\frac{W}{n}\frac{1}{\beta}\right)^{k-1}\frac{1}{V}\left(\alpha^k + \frac{1}{k+1}\left(\frac{S}{2}\right)^k\right) \end{aligned} \tag{15}$$

which decreases with the increase in duty ratio when $k > 1$. This estimation suggests that $k > 1$ is required to explain the change in duty ratio as observed in legged animals. From these considerations, we assume $k = 2$ in the calculation of the optimal locomotor pattern minimizing eq. (12) in the next section.

3. The optimal locomotor pattern

3.1. The optimal locomotor pattern with identical legs

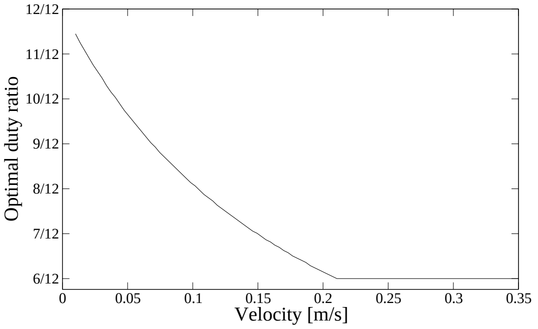

Figure 2: The optimal duty ratio.

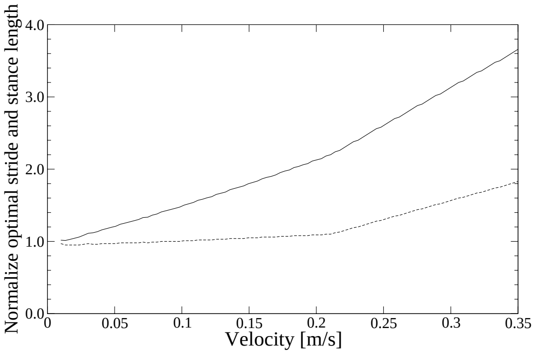

Figure 3: The optimal (solid line) stance and stride length (dashed line). These values are normalized by a leg length.

We calculated the optimal locomotor parameters, duty ratio β and stance length S, to minimize the cost of transport given by eq. (13) for each locomotion velocity. The number of legs are set as $n = 6$, body parameters are described in appendix, and duty ratio is assumed to be larger than 0.5 ($\beta \geq 0.5$) which is the minimum value to enable static locomotion in hexapods. These assumptions only affect the quantitative features in the following results in this section and the lower limit of duty ratio, because the characteristics of all legs are identical and body structure does not affect the estimation of the cost of transport in eq. (13) except the quantitative characteristics caused by body weight W.

Figure 2, 3, and 4 show the relation between the optimal parameters and locomotion velocity. Figure 2 shows that the optimal duty ratio β decreases with velocity as predicted in section 2.3., i.e., the optimal number of stance legs decreases with velocity. Figure 3 shows that the optimal stride length $VT (= S/\beta)$ increases gradually with velocity and the stance length S is almost constant in lower velocities ($v < 0.21$ [m/s]), and they increase with velocity when the duty ratio takes its minimum value ($v > 0.21$ [m/s]). Figure 4 shows that the optimal stride period $T (= S/\beta V)$ decreases with velocity and reaches an almost constant value when the duty ratio takes its minimum value, and that there is no change in the swing duration $(1 - \beta)T$. Figure 5 shows the cost of transport given by the optimal parameters for each velocity. The cost decreases

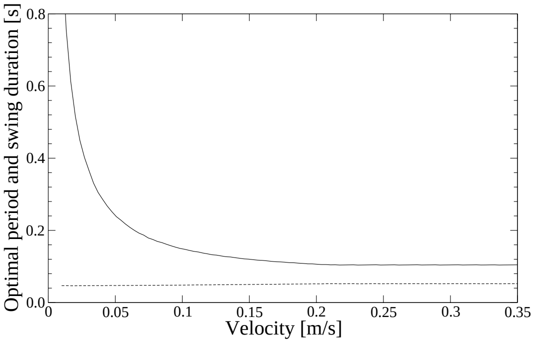

Figure 4: The optimal stride period (solid line) and duration of swing phase (dashed line).

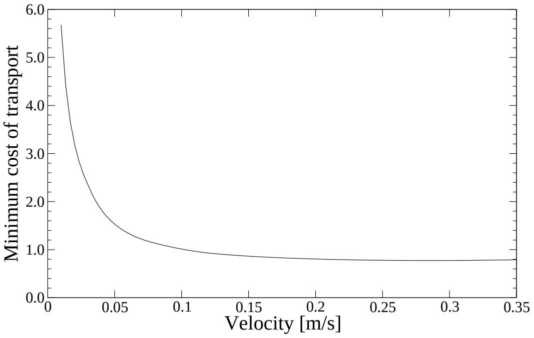

Figure 5: The minimum cost of transport given by the optimal parameters.

with velocities and reaches an almost constant value.

These characteristics of the optimal locomotor pattern well coincident with those of observed legged locomotor patterns as mentioned in Introduction.

3.2. From graded to non-graded gait transition

In the calculation of the optimal locomotor pattern in the previous section ground reaction forces were set as a time-independent constant value, $N_i = W/n\beta$, for all stance legs. However, the ground reaction force for each stance leg changes in real world according to leg motion and the change in the number of stance legs. We recomputed the optimal locomotor pattern from eq. (8), (11), and (12) by considering more realistic distribution of the ground reaction forces.

The number of legs is set as $n = 6$ again, and the order of the leg movement is determined by the rule observed in insects [9]; (1) anti-phase movement of contra-lateral legs in the same segment, (2) forward propagation of the leg movement in ipsi-lateral legs by the same time delay given by a swing duration $(1 - \beta)T$. Based on this rule a gait pattern is defined

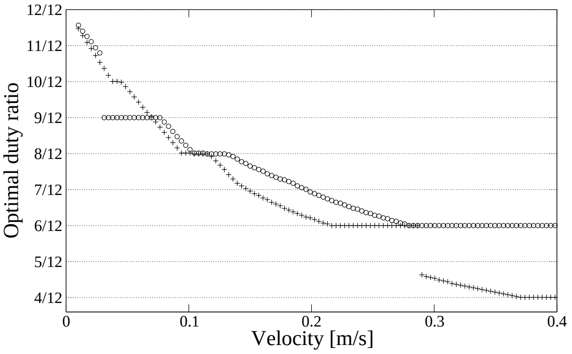

Figure 6: The optimal duty ratio. The pluses and the circles show the optimal duty ratios when the ground reaction forces are given by body weight divided by the number of stance legs and computed by considering the balance of forces around the body, respectively.

for an arbitrary duty ratio and an arbitrary stride period. The ground reaction force for each stance leg is determined for two situations; (1) the ground reaction forces are given by the body weight divided by the number of stance legs, i.e., $N_i = W/n_{st}$, where n_{st} is the number of stance legs, (2) the ground reaction forces are computed by using pseudo-inverse matrix by considering the balance of forces around a rigid body at each moment, in which the variance of the ground reaction forces is minimized. In the former condition no lower limit of duty ratio is assumed, but in the latter condition duty ratio is assumed to be larger than 0.5 ($\beta \geq 0.5$) which enables static locomotions.

Figure 6 shows the optimal duty ratio. The pluses and the circles show the optimal duty ratio when the ground reaction forces are given by $N_i = W/n_{st}$ and computed by using pseudo-inverse matrix, respectively. The optimal duty ratio decreases with velocity but does not change monotonously and tends to take some specific values, e.g., $\beta = 10/12, 8/12, 6/12, 4/12$ in pluses and $\beta = 9/12, 8/12$ in circles, of which values except $\beta = 4/12$ correspond to typical gait patterns in insects reported by Wilson (1966), e.g., wave gait ($\beta \sim 10/12$) and quadrupod gait ($\beta \sim 8/12$), and tripod gait ($\beta \sim 6/12$). The transition between gaits shows not only graded transition but also a discrete transition around $v = 0.03$ [m/s] in circles and around $v = 0.28$ [m/s] in pluses. When we use eq. (13) in the calculation of the cost of transport, that is, the ground reaction forces are equally set as $N_i = W/n\beta$, such non-monotonous transition of the optimal duty ratio is not observed (Fig. 2).

These results suggest that the emergence of specific gait patterns and of discrete gait transition are results of the optimization on the cost of transport, and the change in the number of stance legs at each moment, which causes change in the distribution of the ground reaction forces on stance legs, would be a crucial trigger to cause such non-graded gait transition as suggested in Nishii (2000).

The emergence of different set of the stable duty ratios for different distribution of ground reaction forces suggests that body structure would determine the optimal set of gait patterns, which would explain the facts that different species of insects and different birth stages of some insects show different set of gait patterns [9, 28].

4. Discussion

The characteristics of the obtained optimal locomotor parameters, the duty ratio, the stance and the stride lengths, the stride period, the swing duration, and the cost of transport, are well coincident with those of observed locomotor patterns mentioned in Introduction. These results suggest that the locomotor pattern of legged animals would be well optimized on the energetic cost. Although we treat a leg as a simple one-link system in the estimation of the mechanical work, the characteristics of the obtained locomotor pattern minimizing the estimated cost also coincidents with those given by using inverse-dynamics computation on a six legged dynamical model with two-link legs by Nishii (1998, 2000). These results suggest that eq. (12) and (13) would give an essential account of energetic costs during locomotion qualitatively.

4.1. Estimation of the heat energy loss

In conventional studies of the optimal legged locomotor pattern on the cost of transport, the mechanical work was considered but the cost by heat emission which is an inevitable term in all actuators has been scarcely considered [29, 30, 31]. In this study we showed the optimality of legged locomotor patterns under the assumption that the heat energy loss is proportional to the square of the generated force. Here, one problem arises whether this assumption is valid or not.

The relation between generated force and heat energy loss during locomotion has not been known in physiological studies because of the difficulty of measuring the energetic cost in *in vivo* experiments. Hatze and Buys (1977) predicted the relation in isometric contractions of arm muscles by an analytical study using mathematical muscle models and suggested that the heat energy loss E_h would increase exponentially with the muscle contraction force f, i.e., $E_h(f) \propto \exp(cf) - 1$, ($c$:constant). For small f this relation is approximated as $E_h(f) \propto cf + \frac{(cf)^2}{2!}$. The minimization of the first term would not reproduce the characteristics of observed locomotor patterns by the reason discussed in section 2.2., and the second term would be essential to explain the optimality of legged locomotor patterns.

4.2. Underlying mechanism causing gait transition

Although the phase transition between observed gait patterns have attracted many researchers, most of theoretical studies concerning this topic have paid attention to the neural control system, such as a neural design of a central pattern generator (CPG) [33, 34, 35, 36, 37, 38]. For instance, Kimura et.al (1993) proposed a neural circuit model to reduce the energetic cost for six legged locomotion of insects and showed that specific gait patterns and phase transition between them are emerged by using the neural circuit model, however, it was not proved that the emerged locomotor pattern was not just a result by using a specific neural circuit but a result of the optimization on the energetic cost.

The result of this paper indicates that the existence of the specific gait pattern and the phase transition could be not just a product of constraints in neural controllers but a result of the optimization on energetic cost, which would be also an important cue to investigate the design of neural systems to produce motor commands for a legged locomotion.

Appendix

In the computations in this study we used the following parameters: the total body mass is $M = 3$ [g], the length of the body is $L = 5$ [cm], the leg mass is $m = 0.2$ [g], the leg length is $l = 1$ [cm], and constants are $a = 1$, $\gamma = 10$. Legs are situated at the front, the middle, and the back of the body.

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