An Adaptive Controller for Two Cooperating Flexible Manipulators

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Outline of Presentation

- Cooperating Flexible Manipulators
- Passivity Ideas
- Large Payload Dynamics
- The Adaptive Controller
- Experimental Apparatus and Results
- Conclusions
Adaptive Control of Rigid Manipulators

- Motivation: Mass property uncertainty

- Typical Controller Structure: adaptive feedforward + PD feedback

- Stability established using:
  ⇒ passivity property due to collocation
  ⇒ [problem is “square”]
  ⇒ dynamics are linear in mass properties
Cooperating Flexible Manipulators

Closed-Loop Multibody System
Cooperating Flexible Manipulator Systems: Characteristics

- Nonlinear system
  ⇒ rigid body nonlinearities “plus vibration modes”

- Input actuation and controlled output are noncollocated
  ⇒ Nonminimum phase system
  ⇒ Nonpassive system

- System is “rigidly” overactuated

- Vibration frequencies and/or mass properties may be uncertain
  ⇒ robust and/or adaptive control
Passivity Definitions

$u(t) \xrightarrow{G} y(t)$

Input $u(t)$, Output $y(t)$

$G$ is a general input/output map
$G$ is passive if

$$\int_0^\tau y^T(t)u(t)\,dt \geq 0, \quad \forall \tau > 0$$

$G$ is strictly passive if

$$\int_0^\tau y^T(t)u(t)\,dt \geq \varepsilon \int_0^\tau u^T(t)u(t)\,dt, \quad \varepsilon > 0, \quad \forall \tau > 0$$
Passivity Theorem

If $G$ is passive and $H$ is strictly passive with finite gain, then the closed-loop system is $L_2$-stable:

$$\{u_d, y_d\} \in L_2 \Rightarrow \{y, u\} \in L_2$$
Kinematics

payload position:

\[ \rho = \mathcal{F}_1(\theta_1, q_{e1}) = \mathcal{F}_2(\theta_2, q_{e2}) \]

payload velocity:

\[
\dot{\rho} = J_{1\theta}(\theta_1, q_{1e})\dot{\theta}_1 + J_{1e}(\theta_1, q_{1e})\dot{q}_{1e} \\
= J_{2\theta}(\theta_2, q_{2e})\dot{\theta}_2 + J_{2e}(\theta_2, q_{2e})\dot{q}_{2e}
\]
The joint torques are determined from $\hat{\tau}$:

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} C_1 J_{1\theta}^T \\ C_2 J_{2\theta}^T \end{bmatrix} \hat{\tau}$$

$C_1$ and $C_2$ with $0 < C_i < 1$ and $C_1 + C_2 = 1$ are load-sharing parameters.
**Modified Output**

**μ-tip rate:**

\[
\dot{\rho}_\mu = \mu \dot{\rho} + (1 - \mu) [C_1 J_1 \dot{\theta}_1 + C_2 J_2 \dot{\theta}_2]
\]

**μ-tip position:**

\[
\rho_\mu(t) = \mu \rho(t) + (1 - \mu) [C_1 F_1(\theta_1, 0) + C_2 F_2(\theta_2, 0)]
\]

For \( \mu = 1 \), \( \rho_\mu = \rho \)

For \( \mu = 0 \), \( \rho_\mu = C_1 F_1(\theta_1, 0) + C_2 F_2(\theta_2, 0) \)
Passivity Results

This system is passive for $\mu < 1$ when the payload is large, i.e.,

$$\int_0^\tau \dot{\rho}_\mu^T(t) \hat{\tau}(t) \, dt \geq 0, \quad \forall \tau > 0$$
Large Payload Motion Equations I

Rigid task-space equations:

\[ M_{\dot{\rho}} \ddot{\rho} + C_\rho(\rho, \dot{\rho}) \dot{\rho} = \ddot{\tau} \]

PLUS

Elastic equations consistent with a cantilevered payload.
Large Payload Motion Equations II

Including only the payload mass properties:

\[
\begin{align*}
M\ddot{\nu} + \nu^\otimes M\nu &= P^{-T}(\rho)\hat{\tau} \\
W(\dot{\nu}, \nu, \nu)a
\end{align*}
\]

where

\[
M = \begin{bmatrix}
    m1 & -c^x \\
    c^x & J
\end{bmatrix}, \quad \nu = \begin{bmatrix}
    v \\
    \omega
\end{bmatrix},
\]

\[
\nu^\otimes = \begin{bmatrix}
    \omega^x & O \\
    v^x & \omega^x
\end{bmatrix}, \quad P = \begin{bmatrix}
    C_{M0}(\rho) & O \\
    O & S_{M0}(\rho)
\end{bmatrix}
\]

\(W\) is the regressor.
\(a\) is a column of mass properties.
Note: \(\nu = P(\rho)\dot{\rho}\)
Key Definitions

desired trajectory: \( \{ \rho_d, \dot{\rho}_d, \ddot{\rho}_d \} \)

tracking error:
\[
\tilde{\rho}_\mu = \rho_\mu - \rho_{\mu d}, \quad \rho_{\mu d} = \rho_d
\]

filtered error:
\[
s_\mu = \tilde{\rho}_\mu + \Lambda \tilde{\rho}_\mu, \quad \Lambda = \Lambda^T > 0
\]

If \( s_\mu \in L_2 \), then \( \tilde{\rho}_\mu \to 0 \) as \( t \to 0 \).

body-frame ‘desired’ trajectory:
\[
\nu_d = P(\rho) \dot{\rho}_d
\]

body-frame ‘reference’ trajectory:
\[
\nu_r = \nu_d - P(\rho) \Lambda \tilde{\rho}_\mu
\]
The Adaptive Controller I

control law:

\[ \tau = P^T W(\dot{\nu}_r, \nu_r, \nu) \hat{a}(t) - K_d s_{\mu} \]

\[ = P^T [\hat{M} \dot{\nu}_r + \nu_r \otimes \hat{M} \nu] - K_d [\dot{\tilde{\rho}}_{\mu} + \Lambda \tilde{\rho}_{\mu}] \]

adaptation law:

\[ \dot{\hat{a}} = -\Gamma W^T(\dot{\nu}_r, \nu_r, \nu) P(\rho) s_{\mu}, \]

\[ \Gamma = \Gamma^T > 0 \]
The Adaptive Controller II

\[-P^T W a + \hat{\tau} - \hat{\tau}_d \]

\[-P^T W \hat{a} \]

\[P^T W \]

\[W^T P \]

\[G \]

\[K_d \]

\[\Gamma^1_s \]

\[s_\mu \]
Experimental Apparatus
Closed-Loop Configuration
Mode Shapes
PD Feedback Alone \((C_1 = C_2 = 0.5, \mu = 0.8)\)
Nonadaptive Results \((C_1 = C_2 = 0.5)\)
Adaptive Results

- x-pos (m) error vs. time
- y-pos (m) error vs. time
- z-orientation (rad) vs. time

For different values of $C_1$: $C_1 = 0.25$ and $C_1 = 0.75$. The graphs show the error in position and orientation over time for each value of $C_1$. The fixed parameter graphs and those with different $C_1$ values are indicated with different markers.
Parameter Estimates

mass (kg) vs. time

- estimate
- payload value

c_y (kg-m) vs. time

J_zz (kg-m^2) vs. time
Summary of Presentation

- Passivity-based adaptive control: $\mu$-tip rates + load-sharing

- Adaptive feedforward depends only on “payload equations”

- Robust since passivity depends only on a large payload

- Results exhibit good tracking