

# Synthesis of Dynamics Based Information Processing System of Robot Using Synchronization in the Coupled Arnold Equations

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## Abstract

In this paper, we study on information processing of robot using synchronization in nonlinear dynamical systems. As a nonlinear dynamics used for information processing, we employ the Arnold equation which is known to show the chaotic behaviors of non-compressive perfect fluid. We design a dynamics based information processing system by which periodic motions of robot are controlled, using synchronization in the coupled Arnold equations. Experimental results illustrate the usefulness of the proposed method.

## 1. Introduction

In biological brains and nervous systems, many nonlinear dynamical phenomena like chaos are observed through physiological experiments. It is considered that such nonlinear dynamical phenomena play important roles in the information processing in brains and nervous systems. For example, Freeman showed that the main components of neural activity in olfactory systems of rabbits are chaotic and at times the activities of neurons may come close to a limit cycle by odor inputs. Mathematical models of olfactory systems and pattern recognition using the models have also been studied[1][2]. Chaotic neural networks and those applications are also studied by some researchers[3]-[5].

Dynamics based information processing using such nonlinear dynamical phenomena is expected to be a new approach to a robot intelligence. Okada et al. proposed an information processing that realizes the memorization and generation of the humanoid whole body motion using the nonlinear dynamics with the polynomial configuration[6].

In this paper, we try to develop an information processing system of robot using nonlinear dy-

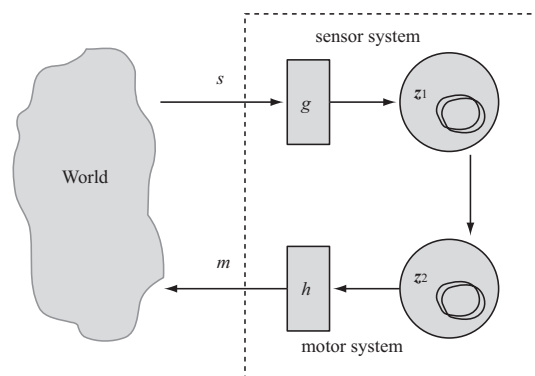


Figure 1: Behavior control using dynamical synchronization

namical phenomena like chaos, transition between chaos and nonchaos, or synchronization. As a nonlinear dynamics used for information processing, we employ the Arnold equation which is known to show the chaotic behaviors of non-compressive perfect fluid. We design a dynamics based information processing system by which periodic motions of robot are controlled, using synchronization in the coupled Arnold equations.

## 2. Behavior Control Using Synchronization of Nonlinear Dynamics

Synchronization phenomena are sometimes observed in two dynamical systems that are connected each other. In this paper, we study on behavior control of robot using such synchronization phenomena or attraction to periodic orbits in nonlinear dynamical systems.

We design a dynamics based information processing system of robot like Fig.1, using a pair

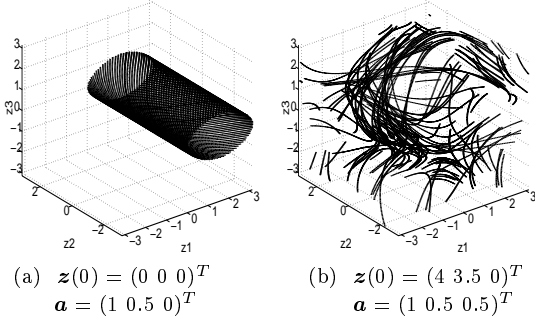


Figure 2: Arnold flow

of dynamical systems connected each other. One of the two dynamical systems is considered as a sensor system, and the other is considered as a motor system. They are mutually connected including environment, and interaction with environment causes synchronization or attraction to limit cycles in the dynamical systems. Behaviors of the robot is controlled or generated using such synchronization or entrainment.

In this paper, we employ the Arnold equation as a nonlinear dynamics and study on such information processing and behavior control of robot as stated above using the Arnold equation. The Arnold equation was used in our study on the chaotic mobile robot[7] and its structure is known. It is a three-dimensional continuous nonlinear dynamical system, and therefore it is comparatively easy to deal with.

### 3. The Arnold Equation

The Arnold equation is written as follows:

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} A_3 \sin z_3 + A_2 \cos z_2 \\ A_1 \sin z_1 + A_3 \cos z_3 \\ A_2 \sin z_2 + A_1 \cos z_1 \end{pmatrix} \quad (1)$$

where  $A_1, A_2$  and  $A_3$  are constant parameters. The Arnold equation is one of steady solutions of 3-dimensional Euler equation, which expresses the behaviors of non-compressive perfect fluids on a 3-dimensional torus space.  $(z_1, z_2, z_3)$  and  $(v_1, v_2, v_3)$  denote the position and velocity of a particle, and  $p$ ,  $(f_1, f_2, f_3)$  and  $\rho$  denote the pressure, external force, and density, respectively. It is known that the Arnold equation shows periodic motion when one of the constants, for example  $A_2$ , is 0 or small (Fig.2(a)), and shows chaotic motion when  $A_2$  is large (Fig.2(b))[8]. Its periodic motions or chaotic motions, and velocity can be varied by

varying its parameters  $A_1, A_2$  and  $A_3$ .

### 4. Information Processing Using Synchronization in the Coupled Arnold Equations

Let  $\mathbf{a} = (A_1 \ A_2 \ A_3)^T$  denote the constant parameter of the Arnold equation, and  $\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z})$  denote the Arnold equation. Since the Arnold equation is a conservative system, the Arnold flow stays in a definite orbit depending upon its initial condition and is not attracted to attractors. Therefore, we design connections between two Arnold equations so that synchronization will occur between the two systems, and design a dynamics based information processing system like Fig.1 using the coupled Arnold equations.

#### 4.1. Synchronization in the coupled Arnold equations

Consider the mutually connected Arnold flows described by

$$\begin{aligned} \dot{\mathbf{z}}_1 &= \mathbf{f}(\mathbf{z}_1) + \mathbf{u}(\mathbf{z}_2) \\ \dot{\mathbf{z}}_2 &= \mathbf{f}(\mathbf{z}_2) + \mathbf{u}(\mathbf{z}_1) \end{aligned} \quad (2)$$

where  $\mathbf{u}(\mathbf{z}_i)$  ( $i = 1, 2$ ) is the input from another Arnold flow.  $\mathbf{u}(\mathbf{z})$  is designed so that  $\mathbf{z}_1$  and  $\mathbf{z}_2$  synchronize in the system.

We design  $\mathbf{u}(\mathbf{z}_i)$  as follows:

$$\mathbf{u}(\mathbf{z}) = kH(\mathbf{z})E\mathbf{v}(\mathbf{z}) \quad (3)$$

$$\mathbf{v}(\mathbf{z}) = (\sin z_1 \ \cos z_1 \ \sin z_2 \ \cos z_2 \ \sin z_3 \ \cos z_3)^T \quad (4)$$

where  $k$  is a positive constant,  $H \in \mathbf{R}^{3 \times 3}$  and  $E$  is

$$E = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \quad (5)$$

$H(\mathbf{z})$  is designed as

$$H = VU^T \quad (6)$$

where  $U$  and  $V$  are matrices obtained from the singular value decomposition (SVD) of  $P$ :

$$P = U\Sigma V^T \quad (7)$$

$$P = \begin{pmatrix} 0 & -\sin z_2 & \cos z_3 \\ \cos z_1 & 0 & -\sin z_3 \\ -\sin z_1 & \cos z_2 & 0 \end{pmatrix} \quad (8)$$

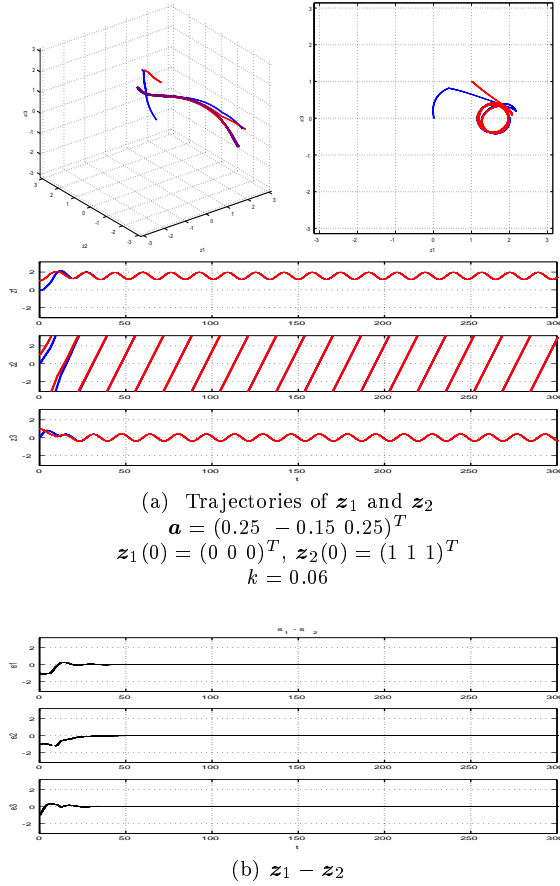


Figure 3: Behavior of the mutually connected Arnold flows

Figure 3 shows an example of behaviors of this system. Figures (a) show trajectories of the two Arnold flows, where the top left figure shows the trajectories in a three-dimensional space, the top right shows a projection of the trajectories to the  $z_1$ - $z_2$  plane, and the bottom shows  $z_1(t)$ ,  $z_2(t)$  and  $z_3(t)$ , respectively. Figure (b) shows  $z_1 - z_2$ . The trajectories of the two Arnold flows are attracted to a periodic pattern and synchronize. The synchronized orbit is dependent upon the parameters. The system has two attracters, and the trajectories are attracted to either of the two attracters. We can use such synchronization phenomena in the mutually connected Arnold flows to control behaviors of a robot, by expanding the mutual connection into the connection including environment as shown in Fig.1.

#### 4.2. Dynamics based information processing system using the coupled Arnold equations

Now we design a dynamics based information processing system of robot using the coupled Arnold equations. Let us consider one of the Arnold flow as a sensor system, and the other as a motor system. Then we cut the connection from the motor system to the sensor system, and re-connect it through the environment as shown in Fig.1. The output from the motor system drives the robot motors through the mapping  $h$ . Sensory signals from the *World* are inputted to the sensor system through the mapping  $g$ . If we can design  $g$  and  $h$  such that the composed mapping  $g \circ (World) \circ h$  becomes an identity mapping, this opened system including the *World* will behave in the same way as the closed, coupled Arnold equations.

Thus, by designing a closed system in which synchronization occurs, we can control behaviors of the opened system. However, in practice it is not easy to design  $g$  and  $h$  so that the mapping  $g \circ (World) \circ h$  becomes an identity mapping, since the *World* has transfer characteristics. It is considered that this complicates the system behaviors.

In this paper, we design  $g$  and  $h$  using neural networks. For periodic motion patterns of the robot and patterns of the coupled Arnold flows, we obtain neural networks  $g$  and  $h$  so that  $g$  maps the robot sensor patterns to the Arnold patterns, and  $h$  maps the Arnold patterns to the robot motion patterns. Using such  $g$  and  $h$ , it is expected that synchronization occurs in the system, and the periodic motions are controlled.

Note that the proposed method provides a framework such that we can separately design synchronized patterns of the dynamical sensor-motor system and motion patterns of the robot.

#### 4.3. Synchronization between two pairs of the sensor-motor system

When applying the proposed method to robots with many DOF, it becomes difficult to get mappings  $g$  and  $h$ . To solve this problem, we consider using some pairs of the sensor-motor system (in the proposed method, the coupled Arnold flows). If there are some pairs of the sensor-motor system, where each pair can control motions of subchains with relatively small number of DOF (for example legs, arms, and so on), we can control motions of the whole robot with many DOF by synchronizing

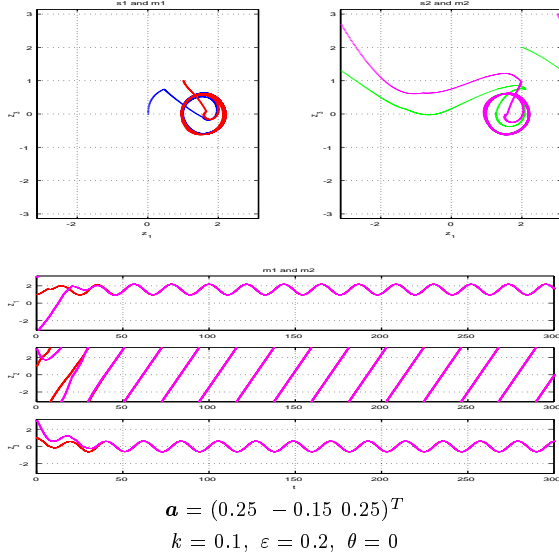


Figure 4: Behavior of two pairs of the mutually connected Arnold flows

the motions of those pairs.

In this section, we consider synchronizing motions of two pairs of the coupled Arnold flows, by interactions between the two pairs. Denote the two pairs of the sensor-motor system as S1-M1 and S2-M2 respectively, state variables of the sensor system and the motor system as  $\mathbf{z}^{si}$ ,  $\mathbf{z}^{mi}$  ( $i = 1, 2$ ), and the Arnold flow as  $\mathbf{f}(\mathbf{z}^i)$ .

First, we consider synchronizing two pairs with the same parameter values, by adding mutual connection between the two motor systems:

$$\begin{aligned}
 \dot{\mathbf{z}}^{si} &= \mathbf{f}(\mathbf{z}^{si}) + \mathbf{u}(\mathbf{z}^{mi}) \\
 \dot{\mathbf{z}}^{mi} &= \mathbf{f}(\mathbf{z}^{mi}) + \mathbf{u}(\mathbf{z}^{si}) + \boldsymbol{\xi}^{mi} \\
 i &= 1, 2
 \end{aligned} \quad (9)$$

where  $\mathbf{u}(\mathbf{z})$  is the connection between the sensor system and the motor system, which was designed in Sect.4.1..  $\boldsymbol{\xi}^{mi}$  denotes the connection between the two motor systems.

Note that, when trajectories of the sensor-motor system designed in Sect.4.1. are synchronized, they flows along one of the coordinate directions. Considering this fact, for example when the two pairs flow along  $z_2$  direction like Fig.3, we determine  $\boldsymbol{\xi}^{mi}$  as follows:

$$\begin{aligned}
 \boldsymbol{\xi}^{m1} &= (0 \quad \varepsilon \sin((z_2^{m2} + \theta) - z_2^{m1}) \quad 0)^T \\
 \boldsymbol{\xi}^{m2} &= (0 \quad \varepsilon \sin(z_2^{m1} - (z_2^{m2} + \theta)) \quad 0)^T \quad (10)
 \end{aligned}$$

Figure 4 shows an example of behaviors of this system. The top figures show projections of the

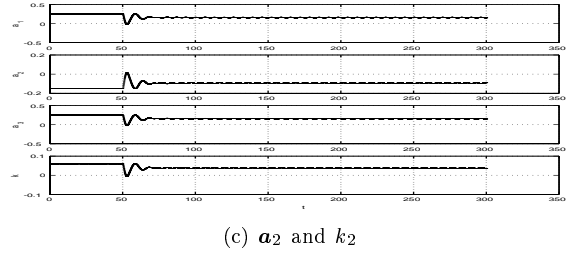
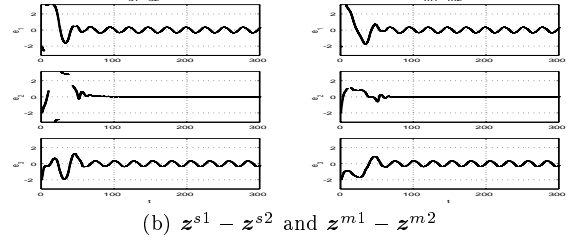
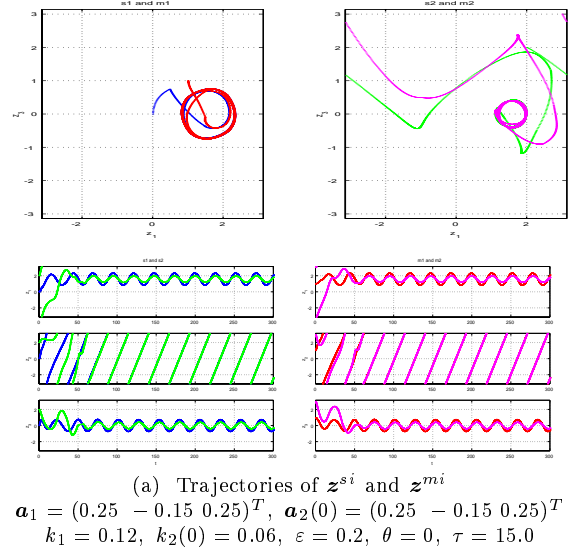


Figure 5: Behavior of two pairs of the mutually connected Arnold flows, with varying parameter values

trajectories to the  $z_1$ - $z_3$  plane, where the top left figure shows the trajectories of the S1-M1 system and the top left shows those of the S2-M2. The bottom figure shows trajectories of M1 and M2. As seen in the figures, motions of the two pairs synchronize.

Next, we consider synchronizing two pairs with different parameter values. When the two pairs have different parameter values, their attracters have different period lengths. Therefore the two pairs are not synchronized by the connection of Eq.(10). In this case we can synchronize them by varying the parameter values, in addition to the connection of Eq.(10). We vary the parameter val-



Figure 6: Robovie

ues of the S2-M2 system as follows:

$$\dot{k}_2 = \frac{1}{\tau} \sin(z_2^{m1} - z_2^{m2}) \quad (11)$$

$$\mathbf{a}_2 = \frac{k_2}{k_2(0)} \mathbf{a}_2(0) \quad (12)$$

where  $k_2$  is the connection parameter of the S2-m2 system,  $\mathbf{a}_2$  is the Arnold equation parameter of the S2-m2 system, and  $\tau$  is a positive constant.  $k_2(0)$  and  $\mathbf{a}_2(0)$  denote initial values of  $k_2$  and  $\mathbf{a}_2$  respectively.

Figure 5 shows an example of behaviors of the system with varying the parameter values. In this example, we added the parameter varying by Eqs.(11) and (12) from  $t = 50$ . Figures (a) show trajectories of the systems. The top figures in (a) show projections of the trajectories to the  $z_1$ - $z_3$  plane, where the top left shows the trajectories of the S1-M1 and the top left shows those of the S2-M2. The bottom left figure in (a) shows trajectories of S1 and S2, and the bottom right shows those of M1 and M2. Figures (b) show  $z^{s1} - z^{s2}$  (left) and  $z^{m1} - z^{m2}$  (right). Figure (c) shows variations of  $\mathbf{a}_2$  and  $k_2$  by Eqs.(11) and (12). As seen in the figures, the period of the S2-M2 comes close to that of the S1-M1, and motions of the two pairs synchronize without change in shape of the synchronized orbits.

## 5. Experimental Results

We have applied the proposed method to a robot and conducted some experiments. We used a robot called Robovie[9], which has two arms ( $4 \times 2$  DOF), a head (3DOF), a mobile platform (2 driving wheels and 1 free wheel), and various sensors (two eyes, skin sensors covering the body, tactile

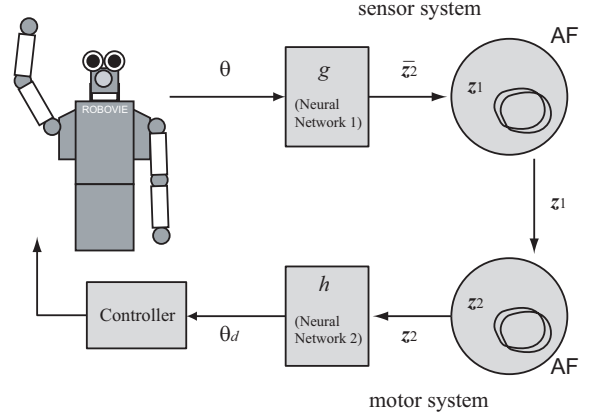


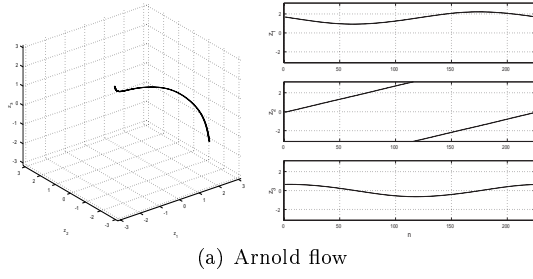
Figure 7: Behavior control of Robovie using dynamical synchronization

sensors around the mobile platform, an omnidirectional vision sensor, two microphones and ultrasonic sensors). Fig.6 shows the Robovie.

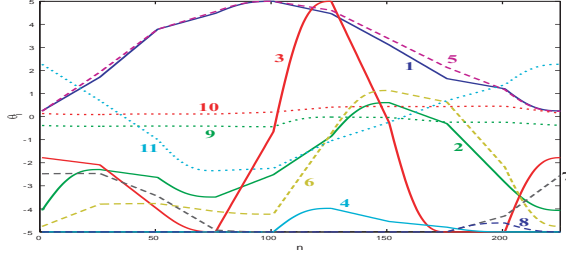
We applied the method proposed in Sect.4. to the Robovie as shown in Fig.7. Periodic motions of the robot are controlled using synchronization in the coupled Arnold equations. In our experiments, we considered controlling 11 DOF motions of the arms and the head. As shown in Fig.7,  $z_2$ , the output from the motor system, is mapped to the reference  $\theta_d$  for the robot motor controller by the mapping  $h$ .  $\theta$ , sensory signal from the potentiometers are mapped to  $z_2$  by  $g$ , and inputted to the sensor system. For the mappings  $g$  and  $h$ , we used the partition nets proposed by MacDorman[10], which realize efficient learning of neural networks.

We designed two pairs of the sensor-motor system, S1-M1 and S2-M2, for the Robovie. The S1-M1 system controls 4 DOF motions of the right arm, and the S2-M2 system controls 7 DOF motions of the left arm and the head. We obtained the mapping  $g_1$  for the S1-M1 system which maps two patterns of periodic robot motion to two patterns of the synchronized Arnold flows, which are shown in Figs.8 (pattern A) and 9 (pattern B). That is,  $g_1$  maps the robot pattern A to the Arnold pattern A, and also maps the robot pattern B to the Arnold pattern B. We also obtained  $h_1$  for the S1-M1,  $g_2$  and  $h_2$  for the S2-M2 in the same way.

In Figs.8 and 9, (a) shows the periodic pattern of the Arnold flow, and (b) shows the pattern of robot motion. Parameters of the coupled Arnold flows in Figs.8(a) and 9(a) were set as follows:

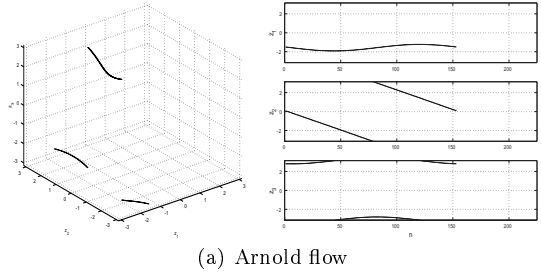


(a) Arnold flow

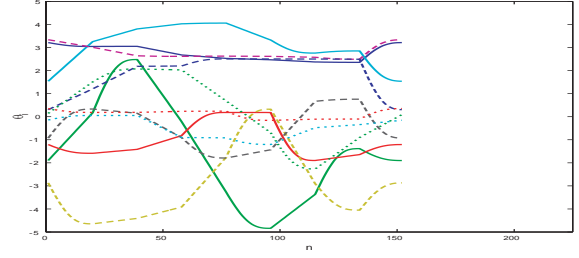


(b) Robovie

Figure 8: Robovie: training data 1 (Pattern A)



(a) Arnold flow



(b) Robovie

Figure 9: Robovie: training data 2 (Pattern B)

$$\begin{aligned} \text{Pat.A : } \mathbf{a} &= (0.25 \quad -0.15 \quad 0.25)^T, \quad k_a = 0.1038 \\ \text{Pat.B : } \mathbf{a} &= (0.25 \quad -0.15 \quad 0.25)^T, \quad k_b = 0.046 \end{aligned}$$

Note that the connection parameter value of Pat.B is different from that of Pat.A. In (b), solid lines show the motions of the right arm joints for the S1-M1 system. Broken lines and dotted lines show the motions of the left arm and the head joints for the S2-M2.

Motions of the Robovie were generated by using synchronizations in the S1-M1 and the S2-M2, and applying the interactions between the two systems. Figs.10 and 11 show an example of generated motions. In this example, the connection parameters in the two systems were set as  $k_1 = k_b$  and  $k_2 = k_a$ . The interaction was applied between M1 and M2, and the parameter varying was applied to the S2-M2 from *step*  $\approx 260$ .

Fig.10(a) shows trajectories of the Arnold flows, where the bottom figure shows trajectories of M1 and M2. Fig.11 shows the resultant robot motion. As seen in the figures, the S1-M1 was attracted to Pat.B and the S2-M2 was attracted to Pat.A respectively. Although the motions of Pat.A and Pat.B had different period lengths, the two systems were successfully synchronized by the parameter varying in addition to the interaction between M1 and M2.

## 6. Conclusion

In this paper, we studied on information processing of robot using synchronization in nonlinear dynamical systems. We designed a dynamics based information processing system by which periodic motions of robot are controlled, using synchronization in the coupled Arnold equations. In the proposed method, we can separately design synchronized patterns of the dynamical sensor-motor system and motion patterns of the robot. Experimental results illustrated the usefulness of the proposed method.

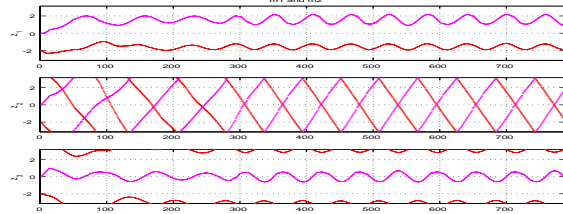
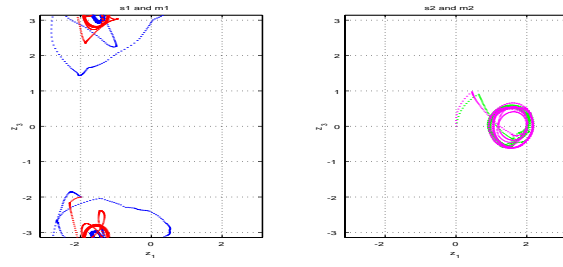
We consider that such dynamics based information processing provides a framework such that the dynamics of body and the dynamics of information processing can be merged, and it would be a basis for a robot intelligence.

## Acknowledgments

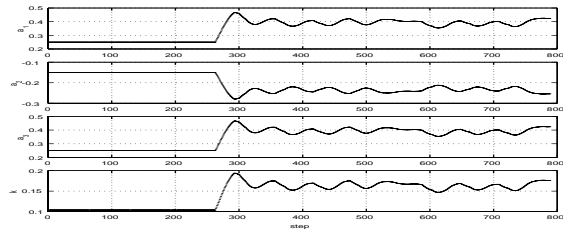
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(a) Trajectories of  $z^{si}$  and  $z^{mi}$



(b)  $a_2$  and  $k_2$

Figure 10: Robovie: resultant trajectories (Arnold flows)

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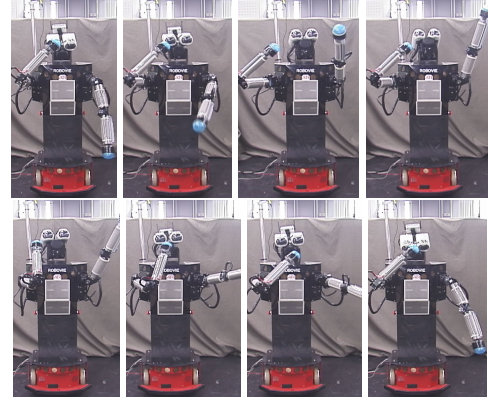
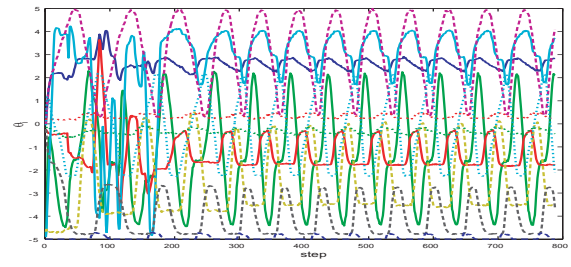


Figure 11: Robovie: resultant trajectories (Robot)

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