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Motion Generate and Control of Quasi-Passsive-Dynamic-Walking based on the concept of Delayed Feedback Control

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Abstract

Recently, Passive-Dynamic-Walking (PDW) has been noticed in the research of a biped walking robot. Then, in this paper, focusing on the entrainment phenomena which is the one of character of PDW, we provide a new control method of quasi-passive-dynamic-walking. Concretely, at first, for the shake of the continuous walking of robot and taking place of the entrainment phenomenon, we adopt the PD gain which is regulated by the state of the contact phase of swing leg. And, considering the making use of the concept of DFC, we use (k-1)-*th* trajectory of the walking robot as the reference trajectory of the k-*th* step. As a result, it can be expected that the robot itself generates the optimum stable trajectory and the walking is stabilized by using this trajectory.

1. Introduction

A lot of researches of humanoid robots or biped locomotion have been carried out now. ASIMO(HONDA) and HRP-series(AIST) are very famous examples. In such researches of walking robots, recently, Passive Dynamic Walking(PDW) which was studied by McGeer[1] at first, have been noticed. As the features of this motion, the following are raised: This walking is very smooth and similar to human's walking and can be realized only by the dynamics of robot without any input torques if the robot walks on smooth slope. Moreover, because of using the effect of gravitational field skillfully, the robot walks with high energy efficiency. From these facts, the various studies of the applications of PDW has been made expecting a realization of a high-efficient and smooth walking of robot [2][3][4][5][6].

Especially, in the research of the application of PDW, some control methods of the Quasi-Passive-Dynamic-Walking, in which the actuators are used just only when the walking begins or disturbances come in, have been proposed [4][5][6]. As one of this control method, Sugimoto *el at.* proposed the control method based on the Delayed Feedback Control(DFC) focusing the contact phase of the swing leg with the ground (they called it Impact Point)[5][6]. Although this control method is very simple and do not require making any reference trajectory in advance, it can not stabilize the walking without an proper initial condition (especially it requires an proper initial velocity) and since it focuses just only on impact point, the performance of stabilization is relatively small and then, it can not stabilize the walking when big disturbances come in.

Then, refering to the weekly guidance control method [4], we consider both making use of the concept of the DFC and providing some reference trajectory for continuous walking. Then, in this paper, we will propose a new control method which can stabilize the robot's walking without an proper initial condition and can stabilize its walking even if some disturbances come in. Here, this control method uses, as k-*th* reference trajectory, (k-1)-*th* step's trajectory of the walking robot, not but a reference trajectory which is made in advance with some simulations. And we adopts the PD gain which is regulated in each steps depending on impact point. According to do so, it is expected that the robot walks continuously and the entrainment phenomena will occur, and then, its walking will converge to the stable trajectory, that is, the trajectory which the robot in PDW generates. This means that the robot walking finally becomes to be stabilized by using the stable reference trajectory which is made by the robot itself.

2. Model of the Walking Robot

A model of the biped robot which we consider is shown in Fig.1.

Let the support leg angle be θ_p , the swing (non-supported) leg angle be θ_w, a slope angle be parameter α,and a torque vector which is supplied to the support leg and the swing leg be $\tau(t) = [\tau_p\ ,\ \tau_w]^T$. And β is the support leg angle at the collision of the swing leg with the ground. Then, the dynamic equation of the robot can be derived using the well known Euler-

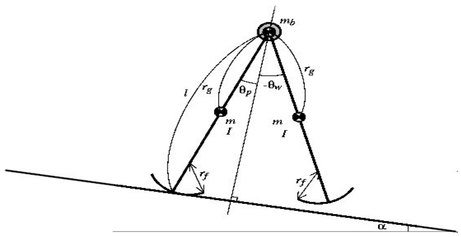

Figure 1: Compass model of Walking robot

Lagrange approach:

$$M(\theta)\ddot{\theta} + N(\theta,\dot{\theta})\dot{\theta} + g(\theta,\alpha) = \tau(t), \quad (1)$$

where $M(\theta)$ is the inertia matrix, $N(\theta,\dot{\theta})\dot{\theta}$ is the centrifugal and Colioris term, and $g(\theta,\alpha)$ is the gravity term. See [4] or [6] in detail. If we assume that a transition of the support leg and the swing leg occurs instantaneously and the impact of the swing leg with the ground is inelastic and occurs without sliding, the equation of transition at the collision of the swing leg with the ground can be derived using the conditions of conservation of angular momentum:

$$P_b(\beta)\dot{\theta^-} = P_a(\beta)\dot{\theta}^+, \quad (2)$$

where $\dot{\theta}^-$,$\dot{\theta}^+$ are the pre-impact and post-impact angular velocities respectively. The details of $P_b(\beta)$, $P_a(\beta)$ are provided in [4] or [6].

And let us introduce the vector p as:

$$p(k) = (\beta_k, \dot{\theta}^-_{p,k}, \dot{\theta}^-_{w,k})^T,$$

where β_k is β at the *k-th* collision, $\dot{\theta}^-_{p,k}$ and $\dot{\theta}^-_{w,k}$ are *k-th* pre-impact angular velocities of the support leg and the swing leg respectively Then we define the p as **Impact point**.

3. Stablity of Passive-Dynamic-Walking

If the input torques are assumed to be constant over each *k-th* step and some assumptions will hold, it can be stated that the discrete dynamic system of impact point: $p(k+1) = \mathcal{P}(p(k),\tau(k))$ can be well defined and the stability of $p(k)$ on this system is equivalent to the stability of PDW of walking robot [5]. Here, expanding this statement, we show that the stability of $p(k)$ on this system is equivalent to the stability of PDW of walking robot even if the input torques are not constant but continue and differentiable between each *k-th* step.

Theorem 1 Let the input torques $\tau(t)$ be continue and differentiable between each *k-th* step. Then, with regard to impact point $p(k)$ and input torques $\tau(t)$, a following map $\mathcal{P}_{cl}$

$$p(k+1) = \mathcal{P}_{cl}(p(k),\tau) \quad (3)$$

can be defined. And, p^* is a stable equilibrium point of this map Eq.(3) with $\tau(t) = 0 \; for \; T_p(k-1) \le t^\forall < T_p(k)$, if and only if, the continuous trajectory of the robot that passes through p^* is stable in the sense of Lyapunov, where $T_p(k)$ is a time when the *k-th* impact occurs.

Proof Basically, it can be proved by similar way of the proof of lemma 1 and 2 in [5]. At first, let the set of the states of the robot just before impact be $\mathcal{S}$, then the target system of the robot can be denoted as follows:

$$\Sigma : \begin{cases} \dot{x}(t) = f_{cl}(x(t)) & (x^-(t) \notin \mathcal{S}) \\ x^+(t) = \Delta(x^-(t)) & (x^-(t) \in \mathcal{S}), \end{cases} \quad (4)$$

where,

$$\begin{aligned} x(t) &:= (\theta_p, \theta_w, \dot{\theta}_p, \dot{\theta}_w)^T, \\ f_{cl} &:= f(x(t)) + g(x(t))\tau(t), \\ f(x(t)) &= \begin{bmatrix} (\dot{\theta}_p, \dot{\theta}_w)^T \\ -M^{-1}(\theta)(N(\theta,\dot{\theta})\dot{\theta} + g(\theta,\alpha)) \end{bmatrix}, \\ g(x(t)) &= \begin{bmatrix} 0 \\ M^{-1}(\theta) \end{bmatrix}. \end{aligned}$$

Because of the condition of $\tau(t)$, it can be said that $f_{cl}(t)$ can have a unique solution which depends continuously on the initial condition between the each *k-th* step, and then, a map $\mathcal{P}_{cl,x}(x,\tau)$ can be well-defined [7]. This map means that the state just before the *k-th* collision x^-_k is mapped to the state just before the (*k*+1)-*th* collision x^-_{k+1} when input torques τ are used. Then, using the following matrixes:

$$E = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, F = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

we can defined a map $\mathcal{P}_{cl}(p(k),\tau)$ as follows:

$$p(k+1) = F\mathcal{P}_{cl,x}(Ep(k),\tau(k)) := \mathcal{P}_{cl}(p(k),\tau). \quad (5)$$

Secondly, because the existence of map $\mathcal{P}_{cl,x}(x,\tau)$ can be shown, using the same way of proof of lemma 2 in [5], we can say that p^* is a stable equilibrium point of the system: $p(k+1) = \mathcal{P}_{cl}(p(k),\tau)$ with $\tau(t) = 0 \; for \; T_p(k-1) \le t^\forall < T_p(k)$, if and only if, the continuous trajectory of the robot that passes through p^* is stable in the sense of Lyapunov.

□

From this theorem, it can be said that even if the input torques are not constant but continue and differentiable between each *k-th* step, the stability of impact point $p(k)$ on the discrete dynamical system is greatly related to the stability of PDW.

4. DFC based Control Method

To propose a new control method of Quasi-Passive-Dynamic-Walking, we particularly consider the following two key ideas. The first one is making use of the concept of DFC so as not to design correctly the reference trajectory which the robot in PDW generates. The second one is providing roughly designed reference trajectory and stabilizing of the walking by using this reference trajectory so as to be possible to start its walking without a proper initial condition or continuous walking even if some disturbances come in.

To construct a controller using the above ideas, in this paper, we focus on the entrainment phenomena which is one of the properties of PDW. The entrainment phenomena of PDW means that even if the robot starts walking with different initial conditions, its walking converges to a specific trajectory which is agree with the trajectory of PDW. However, because the initial condition which can cause PDW exists in very narrow region and PDW is very sensitive to disturbance, it seems that a state of robot which can cause the entrainment phenomena will exist in also narrow region. So, it seems that it is difficult to stabilize Quasi-Passive-Dynamic-Walking only by using the entrainment phenomena.

Then, we construct a new control method which has the next two properties, that is, *"generation of PDW using the entrainment phenomena and the concept of DFC without the correctly design of the reference trajectory of PDW"* and *"stabilization of the walking for the sake of its continuous walking and taking place of the entrainment phenomena"*.

4.1. Our previous control method of Quasi-Passive-Dynamic-Walking

4.1.1. Discrete-DFC based control method

As an example of the control method of Quasi-Passive-Dynamic-Walking, discrete-DFC based control method [5] or [6] can be given. This control method is that, since it can be proved that the stability of PDW is equivalent to the stability of impact point $p(k)$ on the discrete dynamical system:

$$p(k+1) = \mathcal{P}(p(k), \tau), \quad (6)$$

Quasi-Passive-Dynamic-Walking can be stabilized by using the input torques $\tau(k)$ which stabilize $p(k)$ of the system Eq.(6):

$$\begin{aligned} \tau(k) &= K(y(k) - y(k-1)) \\ &= K \begin{bmatrix} P_p(k) - P_p(k-1) \\ P_w(k) - P_w(k-1) \end{bmatrix}, \end{aligned} \quad (7)$$

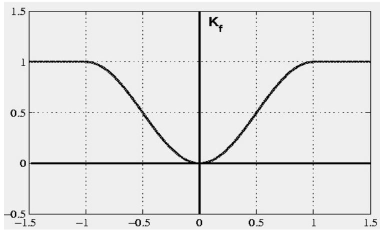

Figure 2: Function K_f at $\gamma = 1.0$

where $P_p(k), P_w(k)$ are the kinetic energy of the support and swing leg at impact point respectively. From Eq.(7), we can see that this control method is very simple and do not need any information of the equilibrium point p^* of Eq.(6), that is, it can stabilize Quasi-Passive-Dynamic-Walking without making any reference trajectory. However, focusing only on impact point, the performance of this control method is relatively not good. So, this can not stabilize the walking when big disturbances come in. Furthermore, this can not stabilize the walking without proper initial conditions especially initial velocities of the legs.

4.1.2. Weekly guidance control method

On the contrary, as one of the control method which utilizes the entrainment phenomena, Osuka and Saruta proposed the following control method [4] (we call it "*weekly guidance* control method"):

$$\begin{aligned} \tau &= K_f(\delta(k))[M(\theta)s + N(\theta)\dot{\theta}^2 + g(\theta, \alpha)], \quad (8) \\ \delta(k) &= \beta(k) - \beta(k-1), \\ s &= \ddot{\theta}_d + K_v(\dot{\theta}_d - \dot{\theta}) + K_p(\theta_d - \theta), \end{aligned}$$

where, $K_f(x)$ is defined by

$$K_f(x) = \begin{cases} 1 & \|x\| \geq \gamma \\ (-cos(\frac{x\pi}{\gamma}) + 1)/2 & \|x\| \leq \gamma, \end{cases} \quad (9)$$

and an example of $K_f(x)$ is shown in Fig.2.

As the feature of this control method, the following can be given: $|\beta_k - \beta_{k-1}|$ is adopted as a evaluate function of the stability of robot's walking and it is used as the weight of trajectory tracking controller. According to the features, even if there is an error between the reference trajectory $r_d = [\theta_d, \dot{\theta}_d]$ used in this control method and the trajectory $r_{id} = [\theta_{id}, \dot{\theta}_{id}]$ which the robot in PDW generates, the trajectory of robot converges to r_{id} and $|\beta_k - \beta_{k-1}|$ becomes small gradually during the robot walks continuously, owing to the entrainment phenomena. And finally, $|\beta_k - \beta_{k-1}|$ becomes zero and then the robot becomes to do PDW. Therefore, it can be expected that

Quasi-Passive-Dynamic-Walking, in which the actuators are used just only when the walking begins or disturbances come in, can be realized by using this control method.

However, we think that there are the following problem in this control method (8).

- In case that the robot's walking is disordered by some disturbances after its walking converges to r_{id}, that is, the robot come to do PDW, is it unreasonable to make the walking to converge to PDW using the r_d once again ? Since the ideal trajectory r_{id} will be made during the robot walks continuously, are there some method of making use of r_{id} for stabilization of its walking ?
- How on earth do we make the reference trajectory r_d ?
- From Section 3., is it better to use the data of impact point $p(k)$ than β_k when the stability of walking is evaluated ?

Especially with regard to r_d, even if there is a difference between r_d and r_{id}, we can expect the walking will converge to r_{id} owing to the entrainment phenomena. But, it is desired that the difference between r_d and r_{id} is as small as possible to improve the efficiency of this control method. As a result, it is needed to make a sufficient proper reference trajectory r_d in advance, and then, it can be said any more that this control method fully makes use of the the entrainment phenomena of PDW.

4.2. The propose control method

From 4.1.1., 4.1.2. and the consequence of Section 3. which means that the stability of impact point $p(k)$ is greatly related to the stability of robot's walking, we propose the following control method.

Updating reference trajectory control method based on DFC

$$\begin{aligned}\tau_k &= K_f(\delta_k)[K_v(\dot{\theta}_{k-1} - \dot{\theta}_k) + \\ &\qquad K_p(\theta_{k-1} - \theta_k)] \quad (10)\\ \delta_k &= ||p(k) - p(k-1)||_\phi,\end{aligned}$$

where θ_k is k-*th* step's $\theta = (\theta_p, \theta_w)^T$, $K_f(\cdot)$ is defined by Eq.(9), ϕ is a constant matrix $\phi \in \mathcal{R}^{3\times3}$ and $||\cdot||_M$ is a norm defined by $||x||_M = \sqrt{x^T M x}$ using a constant matrix $M \in \mathcal{R}^{m\times n}$.

As one of the features of this control method(10), the following can be given: at first, it evaluates the stability of walking by using the data of impact point $p(k)$ and $p(k-1)$. And secondly, it dose tracking control not with r_d which is made in advance but with r_{k-1} which is the (k-1)-*th* trajectory of robot, and as a result, the reference trajectory is updated in each steps.

Since the walking is stabilized by PD-control whose gains are regulated depending on the stability of walking, it can be said that this proposed control method (10) satisfies the specification which is "*stabilization of the walking for the sake of its continuous walking and taking place of the entrainment phenomena*".

And, since updating the reference trajectory using the (k-1)-*th* step trajectory in each steps is equivalent to doing *continuous*-DFC and the entrainment phenomena will cause, we can expect that its walking will converge to r_{id} without making correctly design of the trajectory r_{id} during the robot walks continuously. Therefore, the proposed control method can satisfy the secondary specification which is *"to generate of PDW using the entrainment phenomena and the concept of DFC without the correctly design of the reference trajectory of PDW"*. Moreover, if once the walking of robot converges to PDW, it holds true that $r_k = r_{k-1} = r_{id}$. Then it is also the advantage of this control method that it can use r_{id} as the reference trajectory after the convergence to PDW.

Furthermore, with regard to initial reference trajectory r_0, since it can expected that the robot itself makes the ideal trajectory r_{id} during the robot walks continuously, it is enough to design r_0 roughly with which the robot do not fall down at the beginning of its walking.

Remark In case of using the proposed method Eq.(10) with a real robot, it is more reasonable that we, at first, obtain r_{id-sim} by some simulation using the proposed method with an roughly designed r_0, then we use this r_{id-sim} as r_0 when we actually apply the proposed method to the real robot.

5. Computer Simulation

In this section, we investigate the validity of the proposed control method (10) by several simulations. The table 1 lists the values of the simulation model. The initial condition of the robot is set as $\theta_0(0) = [-0.34, 0.34, 0, 0]^T$ and

$$K_p = \begin{pmatrix} 30 & 0 \\ 0 & 30 \end{pmatrix}, \quad K_v = \begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix},$$

$$\phi = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}, \quad \gamma = 1.0.$$

Table 1: Parameters of the simulation model

l	0.3	m
r_g	0.15	m
r_f	0.1	m
m_b	13.87	kg
m	3.11	kg
I	0	$\text{kg} \cdot \text{m}^2$

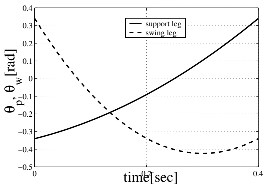

Figure 3: $\theta_{p,0}, \theta_{w,0}$

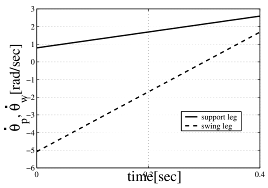

Figure 4: $\dot{\theta}_{p,0}, \dot{\theta}_{w,0}$

The initial reference trajectory $r_0(t)$ is set as follows and shown in Fig.3 and 4:

$$\begin{aligned} \theta_{p,0}(t) &= 2.2667t^2 + 0.79333t - 0.34000, \\ \dot{\theta}_{p,0}(t) &= 4.4894t + 0.79333, \\ \theta_{w,0}(t) &= 8.4524t^2 - 5.0810t - 0.34000, \\ \dot{\theta}_{w,0}(t) &= 16.905t - 5.0810. \end{aligned}$$

This is obtained as following. At first, $\theta_p(t)$ and $\theta_w(t)$ are given as quadratic equations which pass [(0,-0.34),(0.25,0),(0.4,0.34)] and [(0,0.34),(0.3,-0.4),(0.4,-0.34)] respectively. Then, $\dot{\theta}_p(t)$ and $\dot{\theta}_w(t)$ are obtained by differentiating $\theta_p(t)$ and $\theta_w(t)$ respectively. Furthermore, since it can be that k-*th* step's walking period is bigger than (k-1)-*th* step's walking period, we use an 7th polynomial which is approximated to (k-1)-*th* step's trajectory as k-*th* step's reference trajectory.

Simulation results are shown in Fig.5-Fig.7. Fig.5 shows the support leg angle and swing leg angle $\theta_p(t)$ and $\theta_w(t)$, and Fig.6 shows the input torques $\tau(t)$, where the solid line means the support leg and the dotted line means the swing leg respectively. Fig.7 shows the 1,2,3,7 and 24th step's reference trajectory respectively. To compare with our previous control method, the simulation results with weekly guidance control method (8) in which the same r_0 is used as the reference trajectory, are shown in Fig.8 and 9. Fig.8 shows the support and swing leg trajectory and Fig.9 shows the input torques respectively.

From these figures, though the robot's walking is not uniformly and the input torques are needed to continue walking at the beginning of walking, the input torques τ and K_f are decreasing gradually during some steps. And finally, r_k becomes equivalent to r_{k-1} and the robot becomes to walk via PDW. On the contrary,

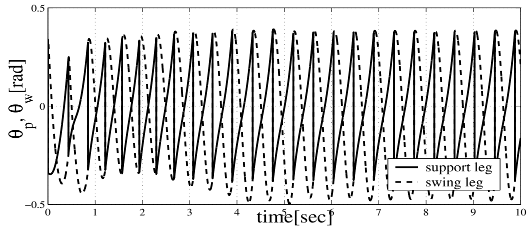

Figure 5: Trajectory of θ_p, θ_w by Eq.(10)

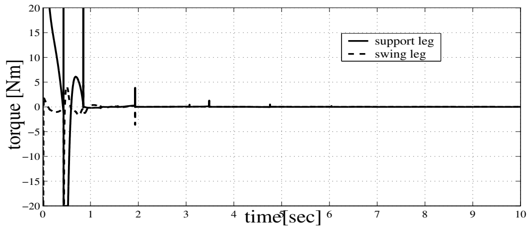

Figure 6: Input torque by Eq.(10)

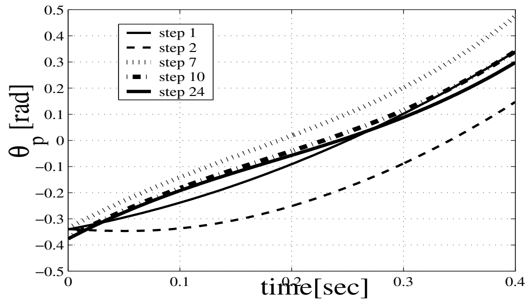

Figure 7: Reference trajectory of θ_p

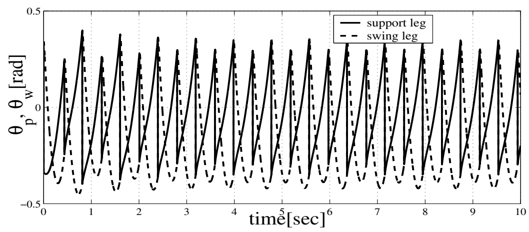

Figure 8: Trajectory of θ_p, θ_w by Eq.(8)

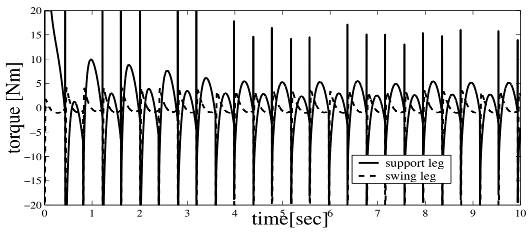

Figure 9: Input torque by Eq.(8)

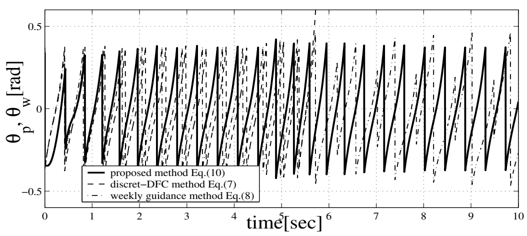

Figure 10: Trajectory of θ_p, θ_w

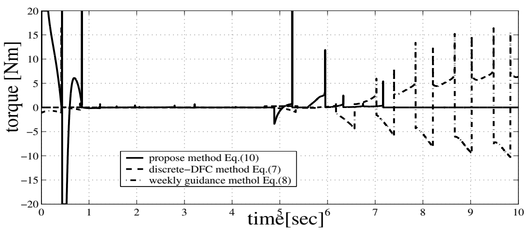

Figure 11: Input torque

the robot's waking with the weekly guidance control Eq.(8) which does not use an proper reference trajectory can not converge to PDW though the robot can continue walking. So, we can say that the proposed control method Eq.(10) works well.

Secondly, we investigate the robustness of the proposed control method against disturbance. Simulation results in which the slope angle α is set as $6[deg]$ (nominal value of α is $3[deg]$) between $5[sec]$ and $6[sec]$, are shown in Fig.10 and 11. To compare with previous control methods, simulation results when Eq.(7) and Eq.(8) are used are also shown in the same figures. Here, there is a problem how we set the reference trajectory r_d in Eq.(8). In this simulation, for easily convergence to PDW, we make the reference trajectory which is very close to the trajectory of PDW but is not agree with PDW's trajectory perfectly.

From these figures, we can see that the robot with the discrete-DFC based control Eq.(7) can not continue walking after the disturbance is added, and the robot with the weekly guidance control Eq. (8) can not converge to PDW again. On the contrary, the robot with the proposed method Eq.(10) can continually walk down after the disturbance is added and becomes to walk via PDW again. So we can say that the proposed method works well.

6. Conclusion and Future Work

In this paper, making use of the concept of DFC and the entrainment phenomena of PDW, we proposed a new control method of Quasi-Passive-Dynamic-Walking. Concretely, the proposed method uses (k-1)-*th* step's trajectory of the walking robot as the reference trajectory of the k-*th* step and regulates the gain according to impact Point. And the effectiveness of the proposed control method was shown through several simulations.

As a problem yet to be solved in the future is to prove the effectiveness of the proposed control method and obtain a method of systematic derivation of K_p, K_v and ϕ in Eq.(10). Furthermore, we should carry out experiments with a real robot.

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