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Control of Snake Like Robot for Locomotion and Manipulation

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Abstract

In the last paper, we proposed a winding control technique using a physical index of horizontal constraint force for a 3D snake-like robot, and it was shown that a winding of a robot can be realized with small joint torque. When it approaches a target point, it is necessary to raise a head and to work like a manipulator. Therefore, a control method of the head configuration using a criterion function, which can be used in both redundant and insufficient number of link cases was proposed. Using a snake like robot called SMA, the validity of the methods was shown experimentally. In this paper control methods in the last paper are summarized and a control method of the winding just before raising the head for reaching to avoid falling down of the whole body. The validity of the proposed method is shown by numerical simulation.

1. Introduction

Environment in which human being works has spread in the various directions by development of technology in recent years. In many cases, such field is accompanied with danger, then the necessity for work robots in such space is increased. Furthermore, the multi functional robot which can do many tasks is desirable in such environment. Therefore, development of the robot which has redundancy was investigated and developed, to name just a few. [1][2][3][4]

A snake like robot is typical example of a robot with redundant degree of freedom, and it can move also in a narrow place and a place with a height difference. Moreover, since it consists of many joints and links, it can change the configuration for specific tasks. Many researcheres discussed possibilities of the usage of such robots for disaster relief or dangerous zone work.[5][6].

As a model of the snake type robot treated in this paper, it is assumed that the robot has passive wheels at each link and a friction coefficient to the tangent direction of the body link is 0 and normal one is infinity, i.e., the snake robots has constraints of not sliding to the normal direction of the wheels. (See Fig. 1)

Figure 1: A snake robot (SMA)

In control of this type of the robot, singular avoidance of postures is one of the important problems where the singular posture means the state when it is impossible for a robot to move further. As typical examples, the shape of a straight line or arc are known.(Fig 2)

For a head position control or a speed control, the posture of the robot is easy to be collapsed into a singular posture like a straight line using so called input-output linearization only for movement of the head position. In order to avoid this problem, the control law using dynamic manipulability was proposed.[5]

In the last paper, we proposed a winding control technique using a physical index of horizontal constraint force. [7] [8] If a shape of robot is like a singular posture, the constraint force becomes very large. By keeping this value small, a winding of a robot can be realized with small joint torque and winding pattern,i.e., spatial frequency and amplitude of the winding, can be easily controlled by a parameter.

When it approaches a target point, it is needed to

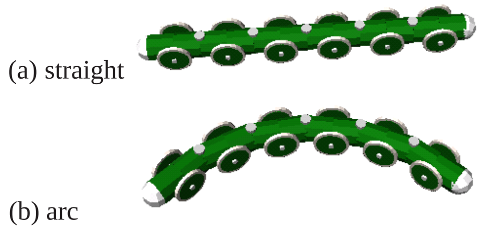


Figure 2: Singular posture

raise a head and to work like a manipulator. However, depending on the number of links to raise, the degree of freedom may be insufficient or conversely redundant. Therefore, a technique of the head configuration control using a criterion function which is not influenced a number of links to raise was proposed.

In order to show the validity of the proposed methods, we constructed a snake like robot called SMA (Super-Mechano Anaconda). Using the experimental system, we showed that the winding pattern with which the robot can avoid a singular posture is generated automatically, and head position and head configuration converge to a desired one. In this paper we summarized the last results and a control method of the winding just before raising the head for reaching to avoid falling down of the whole body. The validity of the proposed method is shown by numerical simulation.

2. Model of a robot

2.1. Model

A model of the robot is shown in Fig.4, and the next assumptions are introduced.

- The robot is multi-link structure which consists of a rigid body.
- Each angular value is relative one, and 0 value means that a robot is like a straight line.
- One module (Fig.3) consists of 2 links. It has a vertical rotation joint at middle, and horizontal one at both sides. All joints are able to be actuated. The considered robot consists of 9 modules.
- A module has passive wheels on the same axis with vertical joint,and it touches a floor with only them.
- The friction force to tangential direction is zero, and that to the normal one is infinity.
- The links before r_F, a middle vertical rotating joint, have 3 dimensional motion, and those after it, have 2 dimensional movement. (Note that the proposed metheds can be also applied for if some joints without wheels are introduced in 2D part with some modifications.)

Refer to the table1 for notations of the robot configuration.

Table 1: Link parameter

notation	definition
l	length of 1 link [m]
θ_h	horizontal rotation angle [rad]
θ_v	vertical rotation angle [rad]
$\boldsymbol{r}_r$	tail position $(\boldsymbol{R}^{3\times1})$
$\boldsymbol{r}_f$	head in plane$(\boldsymbol{R}^{3\times1})$
$\boldsymbol{r}_h$	head position and configuration$(\boldsymbol{R}^{6\times1})$

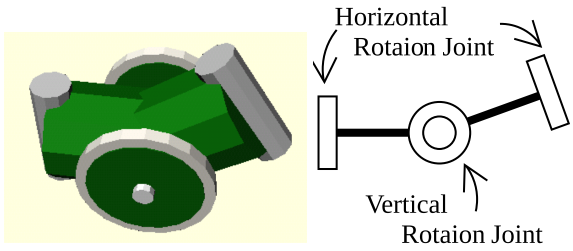


Figure 3: 1 module

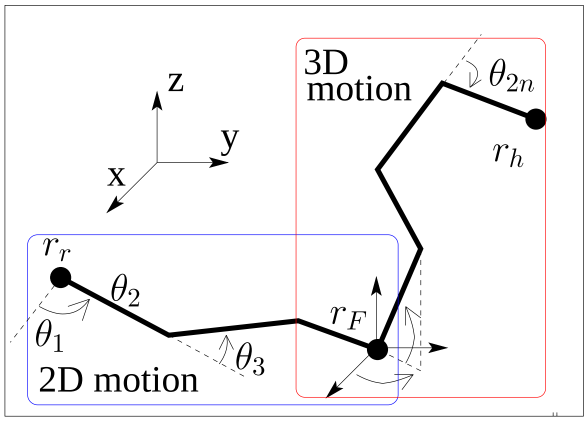


Figure 4: Model

2.2. Motion equation

A generalized coordinates $\boldsymbol{q}$ of the robot consists of tail coordinates $\boldsymbol{r}_r$ and relative angles of links θ as

$$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{r}_r \\ \boldsymbol{\theta} \end{bmatrix}. \tag{1}$$

In description of the movement, the whole motion equation in consideration under the constraint is given by a geometric model as follows :

$$\dot{\boldsymbol{q}} = \boldsymbol{\tau} - \boldsymbol{J}_c^T \boldsymbol{\lambda}, \quad \boldsymbol{\tau} = \begin{bmatrix} 0^{3\times1} \\ \boldsymbol{u} \end{bmatrix}, \tag{2}$$

$$\boldsymbol{J}_c \dot{\boldsymbol{q}} = 0. \tag{3}$$

Although the same argument can be carried out even if we use a torque model, however, in order to simplify the discussion, the geometric model is used here. Even in the geometric model λ is a constraint velocity, however, it is also called constraint force.

2.3. Constraint term

If assumptions are taken into consideration, modules do not slide to the normal direction. This can be expressed as a velocity constraint, and it is expressed as

$$\dot{x}_n \sin\phi_n - \dot{y}_n \cos\phi_n = 0, \quad (4)$$

$$x_n = x_r + 2l\sum_{i=1}^{n-1}\cos\phi_i + l\cos\phi_n \quad (5)$$

$$, y_n = y_r + 2l\sum_{i=1}^{n-1}\sin\phi_i + l\sin\phi_n, \quad (6)$$

$$\boldsymbol{\phi} = \boldsymbol{E\theta} \quad (7)$$

$$= \begin{bmatrix} 1 & & & & & & & & \\ 1 & 0 & 1 & & & & & & \\ 1 & 0 & 1 & 0 & 1 & & & & \\ \vdots & \vdots & & & & & & \ddots & \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & \dots & 1 \end{bmatrix}\boldsymbol{\theta}. \quad (8)$$

where $(\dot{x}_n, \dot{y}_n)$, ϕ_n and $\boldsymbol{E}$ stand for n_th joint coordinate, an absolute angular of each link and transform matrix respectively.

These are substituted for eq.(4),

$$\boldsymbol{A}_\phi\dot{\boldsymbol{\phi}} = \boldsymbol{B}\dot{\boldsymbol{r}}_r \quad (9)$$

$$\boldsymbol{A}_\phi\boldsymbol{E}\dot{\boldsymbol{\theta}} = \boldsymbol{B}\dot{\boldsymbol{r}}_r \quad (10)$$

$$\boldsymbol{A}\dot{\boldsymbol{\theta}} = \boldsymbol{B}\dot{\boldsymbol{r}}_r \quad (11)$$

$$\begin{bmatrix}\boldsymbol{B} & -\boldsymbol{A}\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{r}}_r \\ \dot{\boldsymbol{\theta}}\end{bmatrix} = \boldsymbol{J}_c\dot{\boldsymbol{q}} = 0 \quad (12)$$

and $\boldsymbol{\lambda}$ is calculated by motion equation(2)(3) as follows:

$$\boldsymbol{J}_c(\boldsymbol{\tau} - \boldsymbol{J}_c^T\boldsymbol{\lambda}) = 0 \quad (13)$$

$$\boldsymbol{\lambda} = (\boldsymbol{J}_c\boldsymbol{J}_c^T)^{-1}\boldsymbol{J}_c\boldsymbol{\tau} \quad (14)$$

Therefore the motion equation without the constraint term is given by

$$\dot{\boldsymbol{q}} = \boldsymbol{\tau} - \boldsymbol{J}_c^T(\boldsymbol{J}_c\boldsymbol{J}_c^T)^{-1}\boldsymbol{J}_c\boldsymbol{\tau} \quad (15)$$

$$= (\boldsymbol{I} - \boldsymbol{J}_c^T(\boldsymbol{J}_c\boldsymbol{J}_c^T)^{-1}\boldsymbol{J}_c)\boldsymbol{\tau} =: \boldsymbol{X\tau}, \quad (16)$$

where $\boldsymbol{A}_\phi$, $\boldsymbol{B}$ in eq.(11) are

$$\boldsymbol{A}_\phi = \begin{bmatrix} -l, & & & \\ -2l\cos(\phi_2-\phi_1), & -l & & \\ -2l\cos(\phi_3-\phi_1), & -2l\cos(\phi_3-\phi_2), & -l & \\ \vdots & \vdots & & \\ -2l\cos(\phi_{10}-\phi_1), & -2l\cos(\phi_{10}-\phi_2), & \dots & -2l\cos(\phi_{10}-\phi_9) \; -l \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} -\sin\phi_1, & \cos\phi_1 \\ -\sin\phi_2, & \cos\phi_2 \\ -\sin\phi_3, & \cos\phi_3 \\ \vdots & \\ -\sin\phi_{10}, & \cos\phi_{10} \end{bmatrix}. \quad (17)$$

3. Control law

3.1. Winding control

We will explain about a control law for 2 dimensional winding motion. A fundamental idea is to modify a desired velocity to a part of 2 dimensional motion part $\boldsymbol{r}_F$ in order to reduce the constraint force.

If only desire velocity is given to head $\boldsymbol{r}_F$ (i.e., if input output linearization at head coordinate is used), finally the robot will converge to a singular posture like a straight line. In this case, constraint force becomes very large and it is impossible for the robot to move anymore. Such a situation must be avoided.

For the purpose, the desired velocity $\boldsymbol{\alpha}$ is determined as follows by a liner combination of two vectors $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ for velocity control and reduction of constraint force, respectively:

$$\boldsymbol{\alpha} = w_1\boldsymbol{\alpha}_1 + w_2\boldsymbol{\alpha}_2 \quad (18)$$

where w_1, w_2 are positive weight numbers.

3.1.1. *[Velocity control term : $\boldsymbol{\alpha}_1$]*

The term for velocity control is determined from the difference of an actual velocity of the head in a plane, and desired one. The direction of desire velocity is calculated based on a point that is located in front of the controlled point by L on x axis where x axis is defined as the desired direction(show Fig.5). The velocity $\boldsymbol{\alpha}_1$ is defined as follows:

$$\boldsymbol{\alpha}_1 = \frac{\boldsymbol{v}_d - \boldsymbol{v}}{\|\boldsymbol{v}_d - \boldsymbol{v}\| + \epsilon} \quad (19)$$

where ϵ is small positive real number and $\boldsymbol{\alpha}_1$ is normalized vector.

3.1.2. *[Reduction of constraint force term : $\boldsymbol{\alpha}_2$]*

The constraint force normal to the wheel is equal to the constraint term in the dynamics shown as follows since the constraint Jacobian is consistent to the real constraint force. The amount of movements to the normal direction is represented $\boldsymbol{\xi}$. By the velocity constraint equation of eq.(11), the amount of change $\delta\boldsymbol{\xi}$ can be calculated as

$$\delta\boldsymbol{\xi} = \boldsymbol{A}\delta\boldsymbol{\theta} - \boldsymbol{B}\delta\boldsymbol{r}. \tag{20}$$

Using the principle of virtual work, we have

$$\boldsymbol{f}^T\delta\boldsymbol{\xi} = \boldsymbol{\tau}_c^T\delta\boldsymbol{q} \tag{21}$$

where $\boldsymbol{\tau}_c$, $\boldsymbol{f}$ denote constraint force in q space and constraint force in the space of ξ, respectively. Using eq. 20 and eq. 14, simple calculation shows that

$$\boldsymbol{f} = \boldsymbol{\lambda}. \tag{22}$$

As a control purpose, it is considered to give a desired velocity in the direction which constraint force is decreased. Since the constraint force at $t+\Delta t$ with respect to a change of α at t can be approximated as

$$\begin{aligned}&\|\boldsymbol{f}(\boldsymbol{\alpha}+\delta\boldsymbol{\alpha}, t+\Delta t)\|\\ \simeq\ &\|\boldsymbol{f}(\boldsymbol{\alpha}, t+\Delta t)\| + \frac{\partial\|\boldsymbol{f}(\boldsymbol{\alpha}, t+\Delta t)\|}{\partial\boldsymbol{\alpha}}\delta\boldsymbol{\alpha}.\end{aligned} \tag{23}$$

in order to make the norm smaller after Δt, the amount of change of $\delta\boldsymbol{\alpha}$ is determined as

$$\delta\boldsymbol{\alpha} = -\epsilon\left(\frac{\partial\|\boldsymbol{f}(\boldsymbol{\alpha}, t+\Delta t)\|}{\partial\boldsymbol{\alpha}}\right)^T. \tag{24}$$

Please notice here that this $\delta\alpha$ depends on Δt. Using this equation, norm of constraint force can be made smaller as

$$\|\boldsymbol{f}(\boldsymbol{\alpha}+\delta\boldsymbol{\alpha})\| \simeq \|\boldsymbol{f}(\boldsymbol{\alpha})\| - \epsilon\left\|\frac{\partial\|\boldsymbol{f}(\boldsymbol{\alpha})\|}{\partial\boldsymbol{\alpha}}\right\|^2 \leq \|\boldsymbol{f}(\boldsymbol{\alpha})\|. \tag{25}$$

As mentioned above, the velocity term which decreases the constraint force can be written as follows:

$$\boldsymbol{\alpha}_2 = -\left(\frac{\partial\|\boldsymbol{f}(\boldsymbol{\alpha}_1)\|}{\partial\boldsymbol{\alpha}_1}\right)^T\left(\left\|\frac{\partial\|\boldsymbol{f}(\boldsymbol{\alpha}_1)\|}{\partial\boldsymbol{\alpha}_1}\right\| + \epsilon\right)^{-1} \tag{26}$$

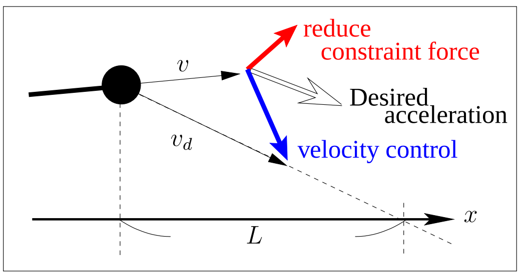


Figure 5: Definition of desire velocity

where $\boldsymbol{\alpha}_2$ is normalized like $\boldsymbol{\alpha}_1$.

Here, velocity control is considered preferentially, and is rewritten $\boldsymbol{\alpha}_2$ as

$$\boldsymbol{\alpha}_2 = -\frac{\|\boldsymbol{v}\|}{\|\boldsymbol{v}_d\|}\left(\frac{\partial\|\boldsymbol{f}(\boldsymbol{\alpha}_1)\|}{\partial\boldsymbol{\alpha}_1}\right)^T\left(\left\|\frac{\partial\|\boldsymbol{f}(\boldsymbol{\alpha}_1)\|}{\partial\boldsymbol{\alpha}_1}\right\| + \epsilon\right)^{-1}. \tag{27}$$

Thereby, it is expected that the influence of a constraint force reduction term is suppressed until real velocity approaches desire one.

3.1.3. [Weight factors: w_1 and w_2]

Weights w_1, w_2 are positive real number, and fundamentally, a winding pattern is controllable only by w_2.

For the determination of control input τ in eq.(16) so that $\dot{r}_f$ is equal to the desired velocity, pseudo inverse of a system gain is used, or it can be determined using a constrained optimization technique so that a norm of the constraint force is minimized while the desired velocity is realized.

3.2. Head configuration control

When a target is approached by the method in the preceding section, it is necessary to raise the head after that and to work like a manipulator, the head position and posture must be controlled. Moreover, depending on the number of links to raise, the degree of freedom may be insufficient or conversely redundant. The effective control method should be considered for both cases.

Therefore, a criterion function L given by eq.(28) is used. By using this function, without depending on the number of links to raise the movement of the head position and posture to the target becomes smooth.

$$L = f(\boldsymbol{x}) + \|\boldsymbol{x}\|^2 + \frac{1}{\epsilon^2}\|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}\|^2 \tag{28}$$

where

$$\begin{cases}\boldsymbol{x} = \boldsymbol{u}\\ \boldsymbol{A} = \boldsymbol{A}_h \quad \boldsymbol{b} = \boldsymbol{b}_h\end{cases} \tag{29}$$

By the motion equation and velocity relationship, we have

$$\begin{cases}\dot{\boldsymbol{q}} = \boldsymbol{X}\boldsymbol{u}\\ \dot{\boldsymbol{r}}_h = \boldsymbol{J}_h\dot{\boldsymbol{q}}.\end{cases} \tag{30}$$

$\boldsymbol{A}_h$ and $\boldsymbol{b}$ are calculated as

$$\begin{aligned}\dot{\boldsymbol{r}}_h &= \boldsymbol{J}_h\dot{\boldsymbol{q}} &(31)\\ &= \boldsymbol{J}_h\boldsymbol{X}\boldsymbol{u} =: \boldsymbol{A}_h\boldsymbol{u} &(32)\\ &= \boldsymbol{u}_d =: \boldsymbol{b}_h &(33)\end{aligned}$$

where $\boldsymbol{J}_h$ is a Jacobian to head position and configuration where $\boldsymbol{b}_r$, and $\boldsymbol{r}_h$, u_d are defined :

$$\boldsymbol{r}_h = \begin{bmatrix} x_h & y_h & z_h & \psi_x & \psi_y & \psi_z \end{bmatrix}^T \quad (34)$$

$$\boldsymbol{u}_d = \dot{\boldsymbol{r}}_d - c_1(\boldsymbol{r}_h - \boldsymbol{r}_d) - c_2 \int (\boldsymbol{r}_h - \boldsymbol{r}_d) dt \quad (35)$$

where $\boldsymbol{r}_d$ is a desire position and configuration, and $\boldsymbol{\psi}$ is angles of roll-pitch-yaw convention, and c_1, c_2 are real numbers.

Eq.(30) shows that $\dot{\boldsymbol{q}}$ is a function of $\boldsymbol{u}$. The optimal input for this criterion function is easily calculated as

$$\boldsymbol{x} = (\boldsymbol{A}^T \boldsymbol{A} + \epsilon^2 \boldsymbol{I})^{-1} \left(\boldsymbol{A}^T \boldsymbol{b} - \frac{\epsilon^2}{2} \frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}} \right). \quad (36)$$

It turns out that the input which minimizes the criterion function L is the function of the real number ϵ. By making this ϵ sufficiently small, a suitable control input is automatically obtained according to the excess and insufficiency of the number of link. (Please refere the details in [7] [8].)

3.3. Preparation for head raising

When the snake robot reaches at a target place, it is supposed to do some work, e.g., manipulating an object using the body or inspecting an environment using eyes. In such situations, the head of the snake robot should be raised and the neck part should be controlled as an manipulator. If the neck part is controlled as an manipulator, reaction forces and moments are exerted to a part touching the ground. If a convex hull composed of the links touching the ground is small, the center of mass of the raised part is easily to move outside the convex hull and the snake robot may loose the balance of the body and may fall down. Once the snake robot stops the forwarding movement, it is very difficult for the robot to increase the area of the convex hull due to the frictions. So it is important to keep the area large enough before raising the head.

In order to enlarge the area, the criterion function to be optimized online used for winding motion is modified as follows:

$$J = \|\boldsymbol{f}(\boldsymbol{\alpha}, t + \Delta t)\| + \frac{c_1}{S(t + \Delta t)} \quad (37)$$

where c_1 is a weighting positive constant, and S is the area of the convex hull which can be calculated as

$$-2S = \sum_{i=1}^{n} (x_{i-1} - x_{i+1}) y_i \quad (38)$$
$$x_0 = x_n, \quad x_{n+1} = x_1,$$

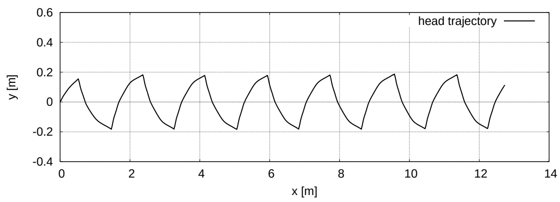
and $(x_i, y_i)(i = 1, \cdots, n)$ are the contact points to the ground, and they are numbered clockwise and (x_0, y_0) is r_F . α_2 is determined to decrease this criterion function.

4. Numerical simulation

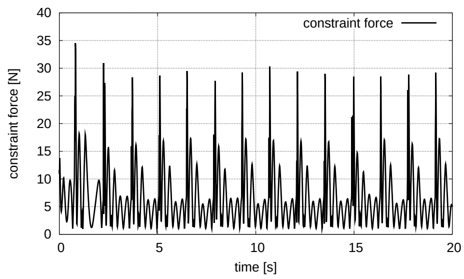
In this section, the validity of the proposed control law of the snake model with the parameter shown in a table 2 is verified in numerical simulations.

4.1. Winding control

From Fig.6, it can be seen that the winding pattern which avoids singular postures is generated automatically, and shows that the norm of constraint force is bounded by a small value. In the figure the generated trajectory of the head is not so smooth, however, the generated one becomes smooth if a dynamic model is used.



(a) a head trajectory in 2D



(b) a norm of constraint force

Figure 6: 2D motion

4.2. Head configuration control

Fig.7 shows that the error $\boldsymbol{e}$ of output values and desire ones,i.e., $\boldsymbol{e} = \boldsymbol{r}_h - \boldsymbol{r}_d$. Initial conditions are set as $\boldsymbol{r}_h = \boldsymbol{0}^{6\times1}$ and desire values are set as constant ones:$\boldsymbol{r}_d = [3.0, 0.0, 0.4, 0.0, 0.0, 0.0]^T$. Using the head configuration control which combined with the winding control, it can be seen that head coordinate converges to a desired one avoiding a singular posture in Fig.7 where the desired position in the direction of x axis is moving at a constant rate of 0.5 [m/sec]. As

Table 2: Parameters of simulation

notation	definition	value
l	length of 1 link [m]	0.08
w_1	a weight of velocity control	0.8
w_2	a weight of constraint force control	0.5
L	parameter for desire velocity	10.0

the validity of the proposed technique was shown by the numerical simulation.

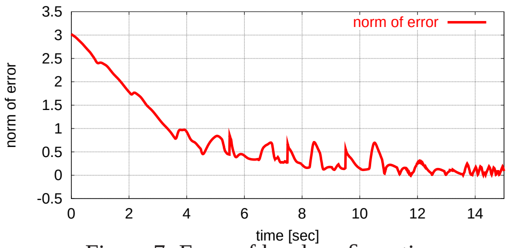


Figure 7: Error of head configuration

4.3. Preparation for head rasing

In order to check the effect of the area term, the snake like robot is controlled using the modified criterion function under the similar condition where the area composed of the first 4 links is taken into account. Fig. 8 shows the responses of the area with the area term and without the area term. As in the figure, it can be observed that the area with modified criterion function converges to a bigger value than that with the original criterion function. In Fig. 9 and Fig. 10, the converged convex hulls by the original criterion function and by the modified criterion function are shown, respectively.

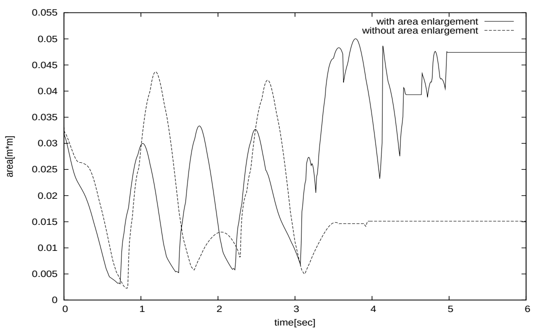


Figure 8: Effect of area term

5. Conclusions

The validity of the proposed control law has been examined in simulations. It can be seen that the winding pattern which avoids singular postures is generated automatically, and the head position converges to desired

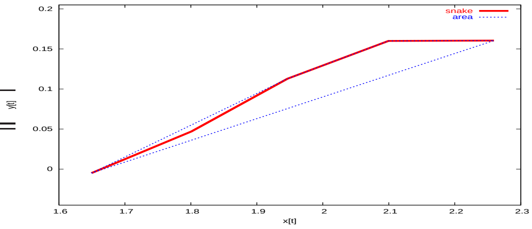


Figure 9: Without area term

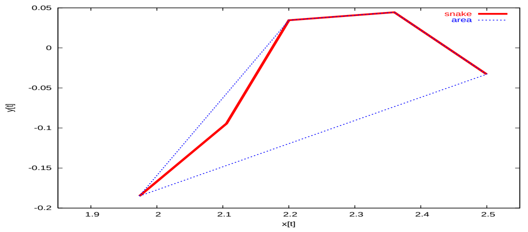


Figure 10: With area term

point. For movies of the experiments, please see an URL:http://www.ctrl.titech.ac.jp/ctrl-labs/yamakita-lab/english/coe/index.html. Future work is to generate various winding pattern to this robot according to environment, and to check these validity.

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