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Minimum Input Walking Gait of Four-DOF Biped Model Solved by Optimal Trajectory Planning Method

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Abstract

In this paper, an optimal trajectory planning study on a planar biped walking mechanism is performed. The biped locomotion mechanism that has thighs, shanks and small feet was modeled as a four-degree-of-freedom (DOF) link system composed of a two-DOF stance leg and a two-DOF swing leg that connected directly at the hip joint. By using the function approximation method and nonlinear programming method, the minimum input walking gaits under full-actuated condition that similar to those of the human walking was obtained. Also, the validity of this method is confirmed by forward dynamic simulation.

1. Introduction

There has long been an interest in understanding human walking locomotion, not only from a desire to build biped mechanisms to perform dangerous tasks instead of humans, but also to improve prosthetic devices for amputees. In recent years, many studies have been published about biped walking systems, either from the viewpoint of constructing human-like biped walking mechanism by joint torque control, or aims to discover walking characteristic by measuring the human motions during locomotion. Mita and Yamaguchi et al. [1], Furusho and Masubuchi [2,3], Kato et al. [4] developed various kinds of biped mechanisms by using model-following control methods to simulate and realize biped locomotion. Vladimir [5] wrote a book describing the methods, technical devices, and procedures used when measuring both pathological and/or healthy human locomotion.

However, most of the researchers did not considered the energy efficiency problem of the walking locomotion, with a notable exception of Silva and Machado [6], Ono and Liu [7]. Silva and Machado characterized the biped motion by a set of locomotion variables, and established the correlation among these locomotion variables and the energy performance. Ono and Liu built a 3-degree-of-freedom (DOF) walking model that composed of a knee-less stance leg and a 2-DOF swing leg. Divided one swing phase into two sections: controlled swing motion of 2-DOF swing leg with straight stance leg and free motion of straight swing leg with straight stance one. And they obtained an energy efficient gait with minimum input torque of the controlled section by using optimal trajectory planning method.

Experimental studies of human locomotion [8] support the hypothesis that the choice of a gait pattern is influenced by energy considerations. That is, the human body will integrate the motions of the various segments and control the activity of the muscles so that the metabolic energy required for a given distance walked is minimized. Bearing these in mind, a trajectory planning study on a 4-DOF planar biped system is performed to give a better insight into the mechanisms of human walking locomotion, which is yet to be fully understood.

The remainder of the paper is organized as follows: A 4-DOF biped model and its motion equations of one step accompany with the cyclic walking conditions derived from the impulse-momentum equations for the toe collision are given in section 2. In section 3, we describe the algorithm used to plan the optimal trajectories of the biped mechanism. Based on these indices, an efficient and natural walking gait in the swing phase is solved by the optimal trajectory planning method, and several numerical results and their forward dynamic confirmation are presented in section 4. Finally, in section 5, we outline the main conclusions and the perspectives towards future research.

2. Biped Walking Mechanism and Its Dynamic Equations

Figure 1 shows the biped mechanism that walks with bent knee. The new model looks a little different from our preceding one. In the present study, we assume that one step of walking is divided into two phases as shown in Fig. 1: a) Swing phase in which one leg is in contact with the ground and the other leg swings forward, b) Stance phase in which the legs trade their roles. In the swing phase (posture 1-2), the stance leg is in contact with the ground and carries the weight of the body from left to right like a 2-DOF inverted pendulum, while the swing leg moves forward like a 2-DOF pendulum in preparation for the next step. The time interval of this phase is denoted by t_1.

The foot exchange takes place instantly (posture2-3) once the swing leg touches the ground. The time interval of this stance phase t_2 is zero. Here, the impact of the swing leg is assumed to be perfectly inelastic while ensuring that no slippage occurs, and an adequate reaction force is prescribed allowing a smooth transition of support. From posture 3, the next swing phase begins. This cyclic pattern of walking movements is repeated over and over, step after step, with a reasonable assumption that successive cycles are all about the same.

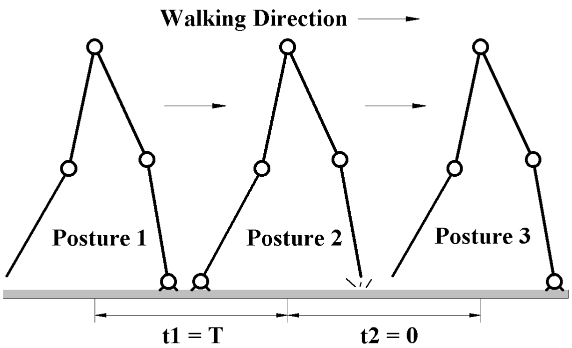


Fig. 1: One step of walking.

Figure 2 shows the planar biped model with the notation used thought this paper. Here we focus only on the biped locomotion in the sagittal plane. The biped mechanism model consists of four links in order to approximate locomotion characteristics similar to those of the lower extremities of the human body (i.e. hip, thigh and shank).

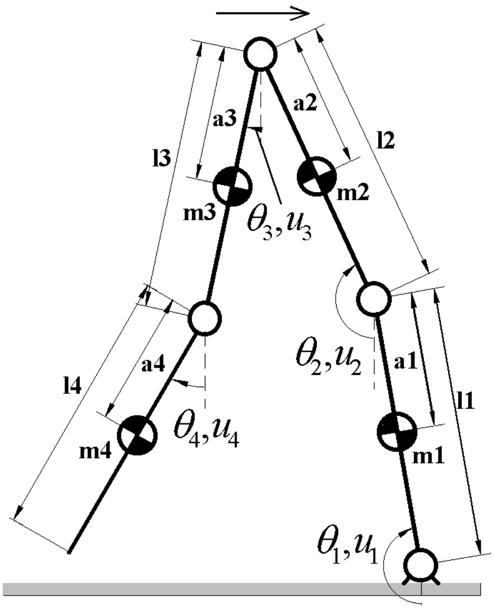


Fig. 2: Planar biped model

Here, we disregarded the torso mass because it has little effect on walking locomotion [7]. And the two legs assumed to be directly connected to each other through a servo-actuator at the hip joint, and both knee and ankle joints are driven by individual servo-actuators. The ankle of the stance leg is modeled as a rotating joint fixed to the ground, while the foot of the swing leg is neglected. Notations l_i, m_i, a_i, I_i, θ_i and u_i in the figure are defined as follows:

l_i : length of link i

m_i : mass of link i

a_i : distance between the mass center of link i and its upper joint

I_i : moment of inertia about the mass center of link i

θ_i : angle of link i

u_i : input torque at joint i

Using Lagrangian dynamics, the dynamic equations for the 4-DOF biped model of the swing phase are derived as follows:

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ & M_{22} & M_{23} & M_{24} \\ & & M_{33} & M_{34} \\ Sym & & & M_{44} \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{Bmatrix} + \tag{1}$$

$$\begin{bmatrix} 0 & C_{12} & C_{13} & C_{14} \\ & 0 & C_{23} & C_{24} \\ & & 0 & C_{34} \\ AntiSym & & & 0 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \\ \dot{\theta}_3^2 \\ \dot{\theta}_4^2 \end{Bmatrix} + \begin{Bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{Bmatrix} = \begin{Bmatrix} u_1 - u_2 \\ u_2 - u_3 \\ u_3 - u_4 \\ u_4 \end{Bmatrix}$$

where M_{ij} , C_{ij} ,and K_i are calculated from l_i, m_i, a_i, I_i and θ_i.

In this biped model, it is assumed that the foot exchange takes place instantly and the collision between the swing leg and the ground is perfectly inelastic for the sake of simplicity. The behavior of the stance phase (posture 2-3) is shown in Fig. 3. $\dot{\theta}_i^-$ represent the angular velocity of link i at the moment right before the foot exchange (posture 2 in Fig.1), whereas $\dot{\theta}_i^+$ stands for the angular velocity of link i right after the foot exchange (posture 3 in Fig.1). By using the impulse-momentum equations for translation and rotation, the relationship between $\dot{\theta}_i^-$ and $\dot{\theta}_i^+$ is obtained as follows:

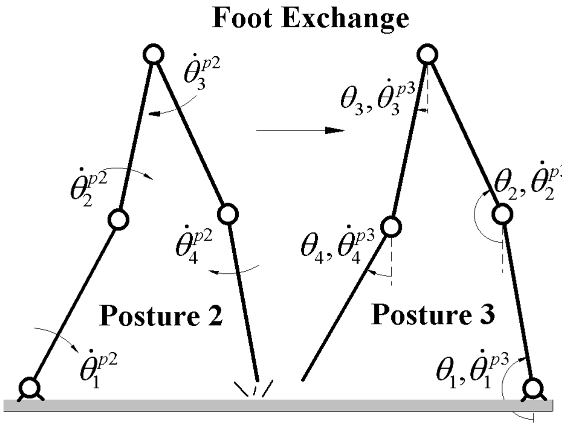


Fig. 3: Stance phase (Foot exchange)

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ & H_{22} & H_{23} & H_{24} \\ & & H_{33} & H_{34} \\ 0 & & & H_{44} \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1^- \\ \dot{\theta}_2^- \\ \dot{\theta}_3^- \\ \dot{\theta}_4^- \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ & Z_{22} & Z_{23} & Z_{24} \\ & & Z_{33} & Z_{34} \\ Sym & & & Z_{44} \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1^+ \\ \dot{\theta}_2^+ \\ \dot{\theta}_3^+ \\ \dot{\theta}_4^+ \end{Bmatrix} \quad (2)$$

where H_{ij} and Z_{ij} are calculated from l_i, m_i, a_i, I_i and θ_i.

From posture 3, the next swing phase begins. With the assumption that the successive step is the same with current step, the motion state variables of posture 3 must be the same as those of posture 1. This can be expressed as:

$$\theta_i^{p3} = \theta_i^{p1} \qquad \dot{\theta}_i^{p3} = \dot{\theta}_i^{p1} \quad (3)$$

From Fig. 3 and Equation (3), the angular relation between posture 2 and posture 1 can be calculated as follows:

$$\begin{aligned} \theta_1^{p2} &= \theta_4^{p1} + \pi \\ \theta_2^{p2} &= \theta_3^{p1} + \pi \\ \theta_3^{p2} &= \theta_2^{p1} - \pi \\ \theta_4^{p2} &= \theta_1^{p1} - \pi \end{aligned} \quad (4)$$

By plugging Equation (3) back into Equation (2), the angular velocities of posture 2 can be expressed by those of posture 1 as follows:

$$\begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{Bmatrix}^{p2} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ & H_{22} & H_{23} & H_{24} \\ & & H_{33} & H_{34} \\ 0 & & & H_{44} \end{bmatrix}^{-1} \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ & Z_{22} & Z_{23} & Z_{24} \\ & & Z_{33} & Z_{34} \\ Sym & & & Z_{44} \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{Bmatrix}^{p1} \quad (5)$$

Equation (4) and (5) is the cyclic condition of walking locomotion and can be rewritten in the general form:

$$c_b(\dot{\theta}^{p1}, \theta^{p2}, \dot{\theta}^{p2}) = 0 \quad (6)$$

3. Optimal Trajectory Planning

In this paper, the swing phase of walking locomotion is solved by the optimal trajectory planning method. Based on the minimized energy hypothesis, the objective function J here is defined as the sum of the integration of the square input torque for one step:

$$J = \sum_{i=1}^{4} k_{wi} \int_0^t u_i^2(t)dt \quad (7)$$

where

k_{wi}: weighting factor corresponding to joint i, here we set $k_{wi} = 1$ for full actuated condition.

u_i: input torque at joint i.

t: time interval of one step.

Rearranging Equation (1), u_i can be expressed as:

$$u_i = \Gamma_i(\theta, \dot{\theta}, \ddot{\theta}) \qquad (i = 1\sim4) \quad (8)$$

where $\theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}^T$.

Applying the function approximation method, the trajectory of the joint angle is represented by a linear combination of the Hermite basis functions [9]. For example, if the swing phase is divided into m subsections, then the joint motion trajectory of the j-th subsection can be expressed as:

$$\theta^j(p,t) = h(t)^T p^j \qquad (j = 1, \dots, m) \quad (9)$$

where $h(t)$ is the Hermite basis function vector and p^j is the boundary state variable vector. In this study, we adopt 7th-order Hermite basis functions and set $m = 1$, thus the p^j is of the form:

$$p^j = \{\theta_b^T, \theta_e^T, \dot{\theta}_b^T, \dot{\theta}_e^T, \ddot{\theta}_b^T, \ddot{\theta}_e^T, \dddot{\theta}_b^T, \dddot{\theta}_e^T\}^T \quad (10)$$

where $\theta_b, \dot{\theta}_b \dots$ are the beginning boundary state variables of the swing phase, while $\theta_e, \dot{\theta}_e \dots$ are the end boundary state variables of the swing phase.

Substituting Equations (8) and (9) into Equation (7), we get an objective function J related to p^j, thus the problem of the calculus of variation subjects to the differential equation constraints is translated into a parameter optimization problem for the basis function coefficients. That is, to find proper unknown boundary state vectors in Equation (10) which minimize the objective function in Equation (7) in a system that is described by Equation (1), and subject to the constraint conditions described by Equation (6). Fig. 4 shows the flowchart of the calculation algorithm.

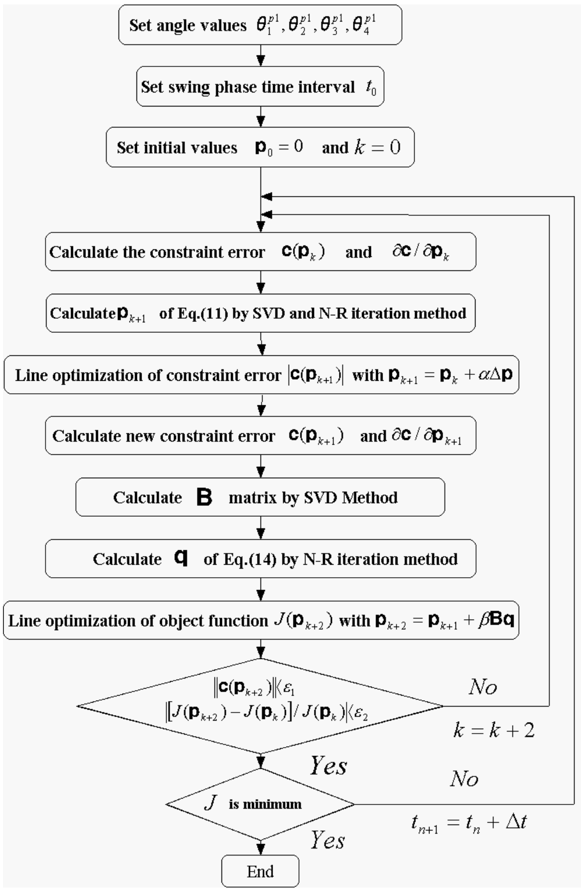


Fig. 4: Flowchart

4. Results and Discussions

Using the trajectory planning method stated above, we got the optimal trajectory walking gaits of the 4-DOF biped model that has the same segment parameters of the lower limb as those of most of us. Table 1 shows the link parameter values used in the calculation.

Table 1. Link parameter values

Parameters	1st Link	2nd Link	3rd Link	4th Link
Length l_i [m]	0.45	0.45	0.45	0.45
Mass m_i [kg]	0.4	0.8	0.8	0.4
Center of mass a_i [m]	0.2	0.15	0.15	0.2
Moment of inertia at mass center I_i [kgm^2]	0.067	0.135	0.135	0.067

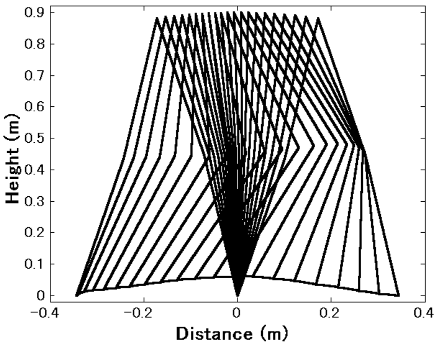
In Fig. 5, we give the optimized trajectory results of the biped model walking through a fixed step length of about 0.35 meters in different time intervals with some knee flexion during stance.

The lowest energy consumption occurs near T=0.60s. We note that the gait of this time interval is very smooth and closely similar to that of human beings. In figures 6 and 7, we give the joint input torque and link angular velocities of each time case respectively. Comparing to other time cases (Fig. 5-7 a, c and d), we can see that the case (b) have the smallest amount and variation both of the joint input torque and the link angular velocities. So, case (b) can be considered to be the most optimal walking gait.

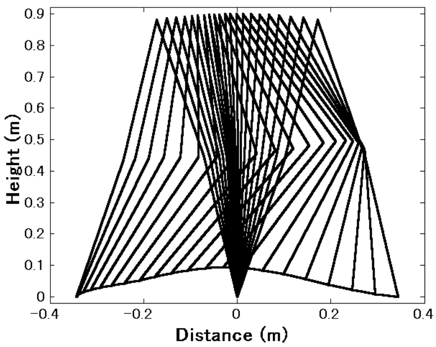
For the purpose of comparing, we also calculated the optimal trajectory gaits of the biped model walking through a same step length of about 0.35 meters in different time intervals without any knee flexion during stance.

This time, the most energy efficient gait occurs near T=0.50s, a little quicker than that of the walking gait with knee flexion. But, the stick figure (Fig. 8a) is not as smooth as the former ones (Fig. 5). And the toe trajectory of the swing leg exhibit somewhat undulation, together with the fluctuation of the joint input torque (Fig. 8b) and link angular velocities (Fig. 8c).

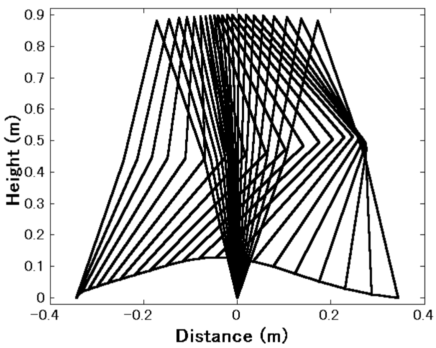
The calculated objective functions J that of the optimized walking locomotion with and without knee flexion during stance are shown in Fig. 9.



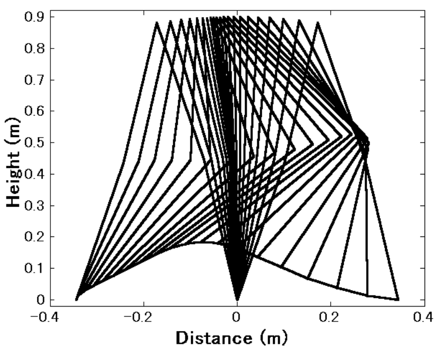
(a) T=0.50 s



(b) T=0.60 s

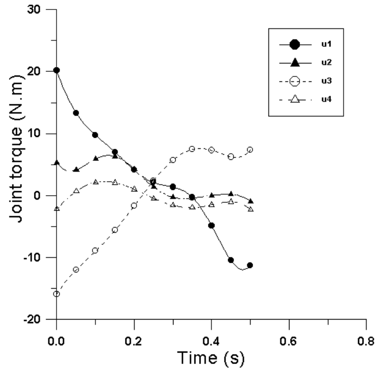


(c) T=0.70 s

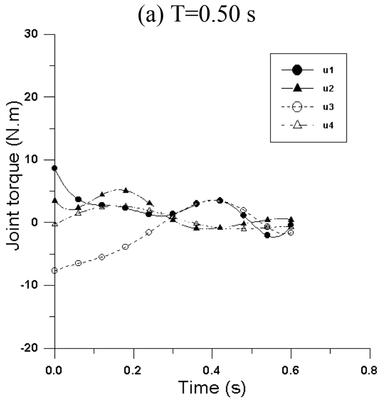


(d) T=0.80 s

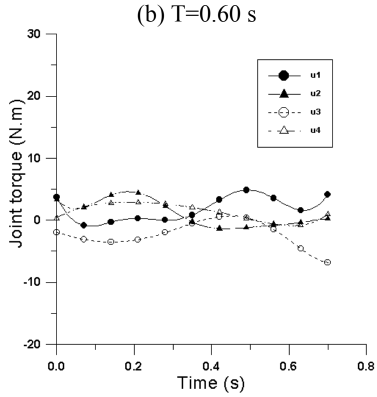
Fig. 5: Stick figures of walking through a distance of about 0.35m with knee flexion in different time intervals



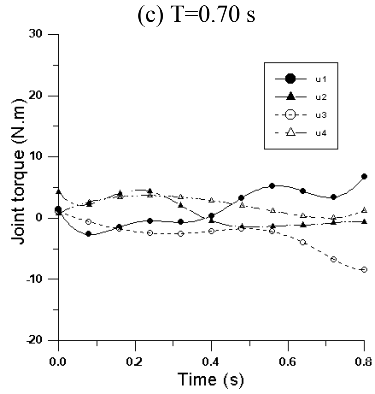
(a) T=0.50 s



(b) T=0.60 s

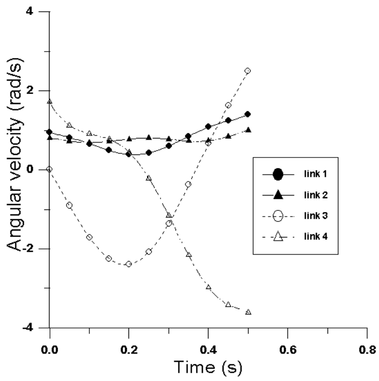


(c) T=0.70 s

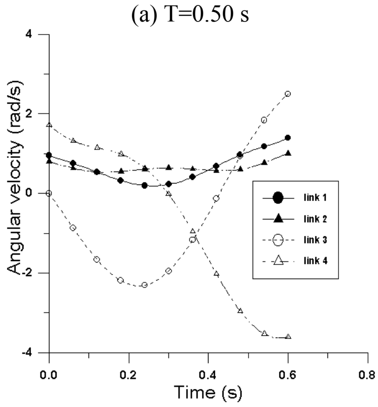


(d) T=0.80 s

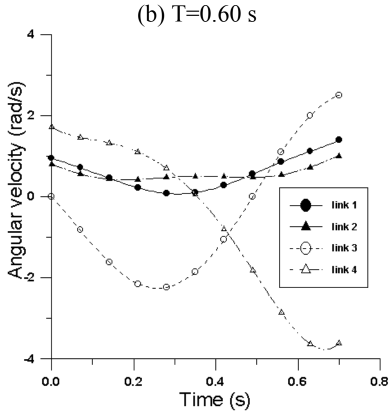
Fig. 6: Input torque of walking with knee flexion. Note that case (b) seems most natural because both the input torque and the leg undulation of this case are smaller than those of the others.



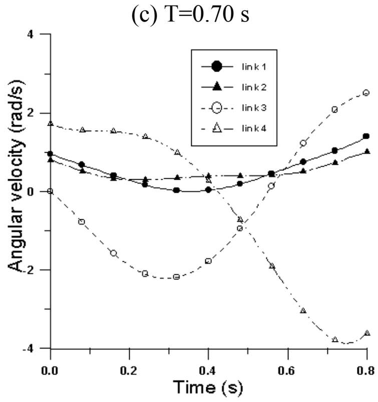
(a) T=0.50 s



(b) T=0.60 s

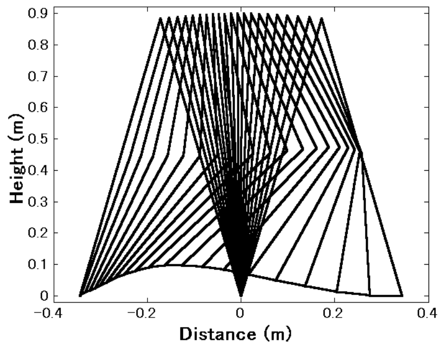


(c) T=0.70 s

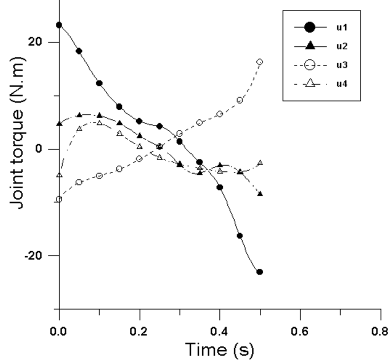


(d) T=0.80 s

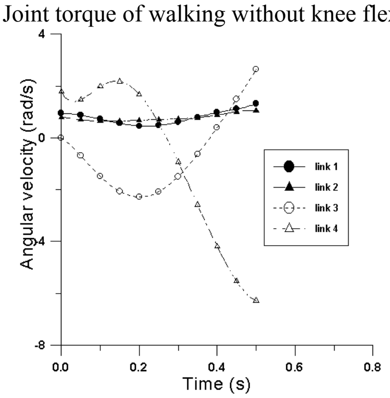
Fig. 7: Angular velocities of walking with knee flexion. Case (b) seems most natural for similar reasons as those of Fig. 6.



(a) Stick figure of walking without knee flexion



(b) Joint torque of walking without knee flexion



(c) Angular velocity of walking without knee flexion

Fig. 8: Walking without knee flexion (T=0.50 s). It seems unnatural because of large joint torque and link angular velocity undulations.

One of our aims of this study is to obtain optimal input torque at all joints of the biped model as shown in Fig.2 to understand the way how human being do it and to realize biped walking mechanisms with energy efficiency. Here comes a question that is whether or not the same biped locomotion can be performed by using the calculated input torque. Adopt the optimized input torque shown in Fig. 6(b) as feed forward control, we simulated the walking motion of the biped, and the results include stick figure (a) and link angular velocities (b) are shown in Fig. 10. It is clear that they are almost all the same with the trajectory planned ones (Fig.5b and Fig.7b).

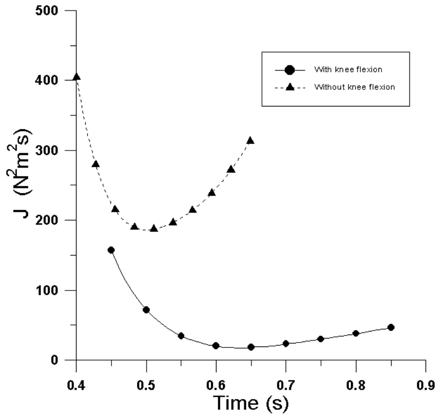
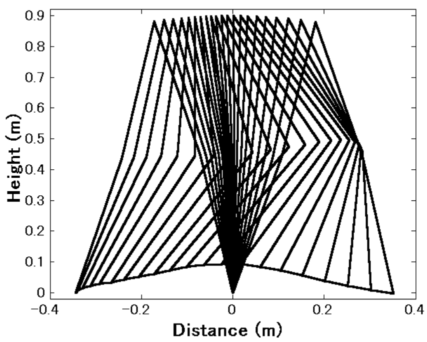
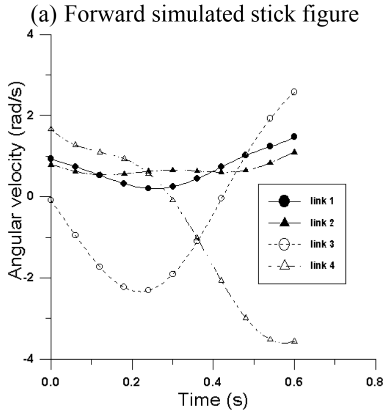


Fig. 9: Calculated objective functions. Note that the knee flexion not only lowered the energy cost, but also lengthened the energy efficient step period.



(a) Forward simulated stick figure



(b) Forward simulated angular velocity

Fig. 10: Forward dynamics simulation results (T=0.60 s). They seem that almost all the same with the trajectory planned ones (Fig. 5b and Fig. 7b).

Although most of us do not have the experience of walking without any knee flexion during stance, from the above figures, especially from the optimized stick figures of the gait with knee flexion, we have to say that it DO seems like the way we walk everyday very much. And the experimental studies of human locomotion [8] demonstrated the fact that there DO have some degrees between the thigh and the shank of the supporting leg during stance.

5. Conclusion

In this paper, the optimal motion trajectory of the 4-DOF biped walking model is numerically calculated by using the trajectory planning method. And the correctness of this method has been proved by forward dynamic simulation. The calculated trajectory seems closely like that of people and confirmed the hypothesis that the choice of a walking gait pattern is influenced by energy considerations. We can say that this optimal trajectory planning method is an effective way to analyze the human walking locomotion.

We will continue the study on human walking by adding torso and feet to the biped model to give a better insight into the principles of human walking locomotion.

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