# Application of Lyapunov Function Based Synthesis of Nonsmooth Limit Cycles to Motion Generation for Humanoid Robots

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## 1. Introduction

In this paper, we propose a Lyapunov function based method of periodic motion generation and a method of dynamic transition between motions of humanoid robots. We calculate a Lyapunov function which keeps constant on a periodic motion and obtain a desired system. The designed system has a stable limit cycle corresponding to a periodic motion, and it is easy to change the stability of the limit cycle. The transient behavior between motions is achieved by destabilizing the current motion and stabilizing the next motion.

## 2. Generation of Smooth Motions

Generally, it is difficult to treat the motion data without any reduction because the humanoid robot has high degree-of-freedom. So first, we reduce the motion data to a lower-dimensional data. We apply a reduction method using singular value decomposition [1]. By approximating the reduced motion data to Fourier series expansion, we obtain a Lyapunov function. The desired system is constructed by the Lyapunov function. Our proposed method is based on [2].

### 3. Generation of Nonsmooth Motions

In this section, we propose a synthesis of system which generates a nonsmooth periodic motion. For simplicity, to generate a nonsmooth motion we consider a piecewise quadratic Lyapunov function [3]. The reduced space is split into several regions and the periodic motions are approximated by an ellipsoidal curve in each region. By constructing subsystems corresponding to each ellipsoidal curve and switching those subsystems, nonsmooth motion pattern can be realized.

### 4. Design of Whole Body Motions

In this section, we design the humanoid whole body motion. The proposed system has a form of  $\dot{y} = f(y)+g(y)$ , and it can be easily destabilized by changing g(x) to -g(x). When we consider whole body mo-

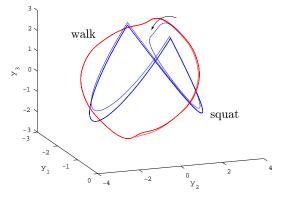


Figure 1: Transition of motion from "walk" to "squat".

tion in which there exists multiple motion patterns this property is useful to let trajectories diverge from the current motion and converge the next motion (Figure 1).

### 5. Conclusions

We have proposed a design method for a nonlinear system which generates a periodic motion of humanoid robots. Generation of smooth and nonsmooth periodic motions are discussed. Finally, the humanoid whole body motin is generated by combining designed systems. The transient behavior of motion is avhieved by changing the stability of periodic motions gradually.

### References

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