# An Adaptive Controller for Two Cooperating Flexible Manipulators

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# Outline of Presentation

- Cooperating Flexible Manipulators
- Passivity Ideas
- Large Payload Dynamics
- The Adaptive Controller
- Experimental Apparatus and Results
- Conclusions

# Adaptive Control of Rigid Manipulators



- Motivation: Mass property uncertainty

- Typical Controller Structure:
  adaptive feedforward + PD feedback

- Stability established using:
  $\Rightarrow$ passivity property due to collocation
  $\Rightarrow$ [problem is "square"]
  $\Rightarrow$ dynamics are linear in mass properties


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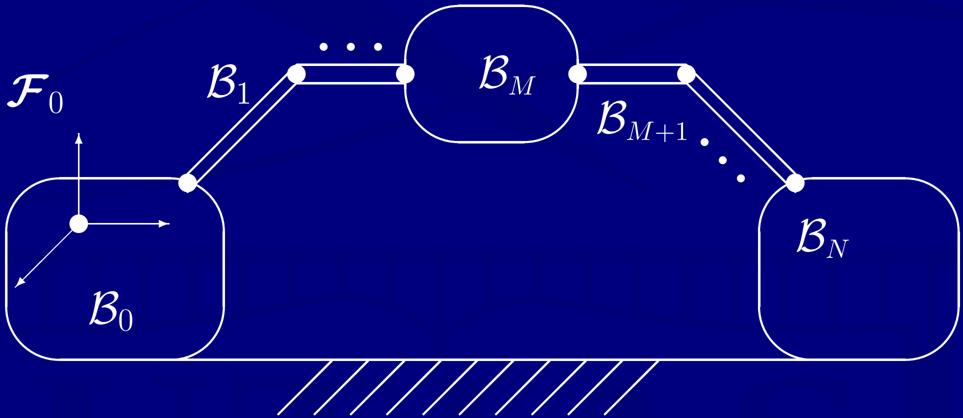
# Cooperating Flexible Manipulators






Closed-Loop Multibody System

# Cooperating Flexible Manipulator Systems: Characteristics



- Nonlinear system

  $\Rightarrow$ rigid body nonlinearities “plus vibration modes”

- Input actuation and controlled output are noncollocated

  $\Rightarrow$ Nonminimum phase system

  $\Rightarrow$ Nonpassive system

- System is “rigidly” overactuated

- Vibration frequencies and/or mass properties may be uncertain

  $\Rightarrow$ robust and/or adaptive control

# Passivity Definitions

$$\underset{\text{input}}{\boldsymbol{u}(t)} \longrightarrow \boxed{\boldsymbol{G}} \longrightarrow \underset{\text{output}}{\boldsymbol{y}(t)}$$

***G*** is a general input/output map

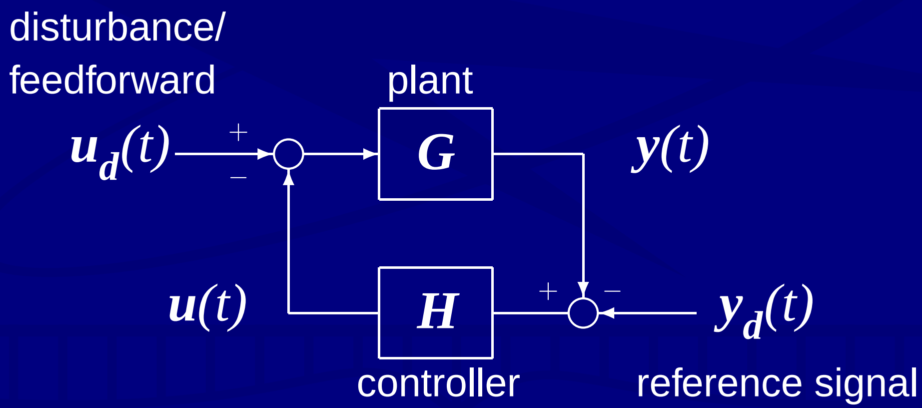
***G*** is **passive** if

$$\boxed{\int_0^\tau \boldsymbol{y}^T(t)\boldsymbol{u}(t)\,dt \geq 0\;, \quad \forall \tau > 0}$$

***G*** is **strictly passive** if

$$\boxed{\begin{gathered}\int_0^\tau \boldsymbol{y}^T(t)\boldsymbol{u}(t)\,dt \;\geq\; \varepsilon\int_0^\tau \boldsymbol{u}^T(t)\boldsymbol{u}(t)\,dt, \\ \varepsilon > 0\;, \quad \forall\,\tau > 0\end{gathered}}$$

# Passivity Theorem




If $\boldsymbol{G}$ is passive and $\boldsymbol{H}$ is strictly passive with finite gain, then the closed-loop system is $L_2$-stable:

$$\{\boldsymbol{u}_d, \boldsymbol{y}_d\} \in L_2 \Rightarrow \{\boldsymbol{y}, \boldsymbol{u}\} \in L_2$$

# Kinematics



payload position:

$$\boldsymbol{\rho} = \boldsymbol{\mathcal{F}}_1(\boldsymbol{\theta}_1, \mathrm{q}_{e1}) = \boldsymbol{\mathcal{F}}_2(\boldsymbol{\theta}_2, \mathrm{q}_{e2})$$

payload velocity:

$$\begin{aligned} \dot{\boldsymbol{\rho}} &= \boldsymbol{J}_{1\theta}(\boldsymbol{\theta}_1, \mathrm{q}_{1e})\dot{\boldsymbol{\theta}}_1 + \boldsymbol{J}_{1e}(\boldsymbol{\theta}_1, \mathrm{q}_{1e})\dot{\mathrm{q}}_{1e} \\ &= \boldsymbol{J}_{2\theta}(\boldsymbol{\theta}_2, \mathrm{q}_{2e})\dot{\boldsymbol{\theta}}_2 + \boldsymbol{J}_{2e}(\boldsymbol{\theta}_2, \mathrm{q}_{2e})\dot{\mathrm{q}}_{2e} \end{aligned}$$

# Modified Input



The joint torques are determined from $\widehat{\boldsymbol{\tau}}$:

$$\boldsymbol{\tau} = \begin{bmatrix} \boldsymbol{\tau}_1 \\ \boldsymbol{\tau}_2 \end{bmatrix} = \begin{bmatrix} C_1 \boldsymbol{J}_{1\theta}^T \\ C_2 \boldsymbol{J}_{2\theta}^T \end{bmatrix} \widehat{\boldsymbol{\tau}}$$

$C_1$ and $C_2$ with $0 < C_i < 1$ and $C_1 + C_2 = 1$ are *load-sharing parameters.*

# Modified Output



$\mu$-tip rate:

$$\dot{\boldsymbol{\rho}}_\mu = \mu\dot{\boldsymbol{\rho}} + (1-\mu)[C_1\boldsymbol{J}_{1\theta}\dot{\boldsymbol{\theta}}_1 + C_2\boldsymbol{J}_{2\theta}\dot{\boldsymbol{\theta}}_2]$$

$\mu$-tip position:

$$\boldsymbol{\rho}_\mu(t) \doteq \mu\boldsymbol{\rho}(t) + (1-\mu)[C_1\boldsymbol{\mathcal{F}}_1(\boldsymbol{\theta}_1,\mathbf{0}) + C_2\boldsymbol{\mathcal{F}}_2(\boldsymbol{\theta}_2,\mathbf{0})]$$

For $\mu = 1$, $\boldsymbol{\rho}_\mu = \boldsymbol{\rho}$
For $\mu = 0$, $\boldsymbol{\rho}_\mu \doteq C_1\boldsymbol{\mathcal{F}}_1(\boldsymbol{\theta}_1,\mathbf{0}) + C_2\boldsymbol{\mathcal{F}}_2(\boldsymbol{\theta}_2,\mathbf{0})$

# Passivity Results



$$\underset{\text{input}}{\widehat{\boldsymbol{\tau}}(t)} \longrightarrow \boxed{\;G\;} \longrightarrow \underset{\text{output}}{\dot{\boldsymbol{\rho}}_{\mu}(t)}$$

This system is passive for $\mu < 1$ when the payload is large, i.e.,

$$\int_0^{\tau} \dot{\boldsymbol{\rho}}_{\mu}^{T}(t)\widehat{\boldsymbol{\tau}}(t)\,dt \geq 0\,, \quad \forall \tau > 0$$

# Large Payload Motion Equations I



Rigid task-space equations:

$$\boldsymbol{M}_{\rho\rho}\ddot{\boldsymbol{\rho}} + \boldsymbol{C}_{\rho}(\boldsymbol{\rho}, \dot{\boldsymbol{\rho}})\dot{\boldsymbol{\rho}} = \widehat{\boldsymbol{\tau}}$$

PLUS

Elastic equations consistent with a cantilevered payload.

# Large Payload Motion Equations II



Including only the payload mass properties:

$$\underbrace{\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{\nu}^{\otimes}\boldsymbol{M}\boldsymbol{\nu}}_{\boldsymbol{W}(\dot{\boldsymbol{\nu}},\boldsymbol{\nu},\boldsymbol{\nu})\mathbf{a}} = \boldsymbol{P}^{-T}(\boldsymbol{\rho})\widehat{\boldsymbol{\tau}}$$

where

$$\boldsymbol{M} = \begin{bmatrix} m\mathbf{1} & -\mathbf{c}^{\times} \\ \mathbf{c}^{\times} & \mathbf{J} \end{bmatrix}, \quad \boldsymbol{\nu} = \begin{bmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{bmatrix},$$

$$\boldsymbol{\nu}^{\otimes} = \begin{bmatrix} \boldsymbol{\omega}^{\times} & \mathbf{O} \\ \mathbf{v}^{\times} & \boldsymbol{\omega}^{\times} \end{bmatrix}, \quad \boldsymbol{P} = \begin{bmatrix} \mathrm{C}_{M0}(\boldsymbol{\rho}) & \mathbf{O} \\ \mathbf{O} & \mathrm{S}_{M0}(\boldsymbol{\rho}) \end{bmatrix}$$

$\boldsymbol{W}$ is the regressor.
$\mathbf{a}$ is a column of mass properties.
Note: $\boldsymbol{\nu} = \boldsymbol{P}(\boldsymbol{\rho})\dot{\boldsymbol{\rho}}$

# Key Definitions



desired trajectory: $\{\boldsymbol{\rho}_d, \dot{\boldsymbol{\rho}}_d, \ddot{\boldsymbol{\rho}}_d\}$
tracking error:

$$\widetilde{\boldsymbol{\rho}}_\mu = \boldsymbol{\rho}_\mu - \boldsymbol{\rho}_{\mu d}, \quad \boldsymbol{\rho}_{\mu d} \doteq \boldsymbol{\rho}_d$$

filtered error:

$$\mathbf{s}_\mu = \dot{\widetilde{\boldsymbol{\rho}}}_\mu + \Lambda \widetilde{\boldsymbol{\rho}}_\mu, \quad \Lambda = \Lambda^T > \mathbf{O}$$

If $\mathbf{s}_\mu \in L_2$, then $\widetilde{\boldsymbol{\rho}}_\mu \to \mathbf{0}$ as $t \to 0$.
body-frame 'desired' trajectory:

$$\boldsymbol{\nu}_d = \boldsymbol{P}(\boldsymbol{\rho})\dot{\boldsymbol{\rho}}_d$$

body-frame 'reference' trajectory:

$$\boldsymbol{\nu}_r = \boldsymbol{\nu}_d - \boldsymbol{P}(\boldsymbol{\rho})\Lambda\widetilde{\boldsymbol{\rho}}_\mu$$

# The Adaptive Controller I



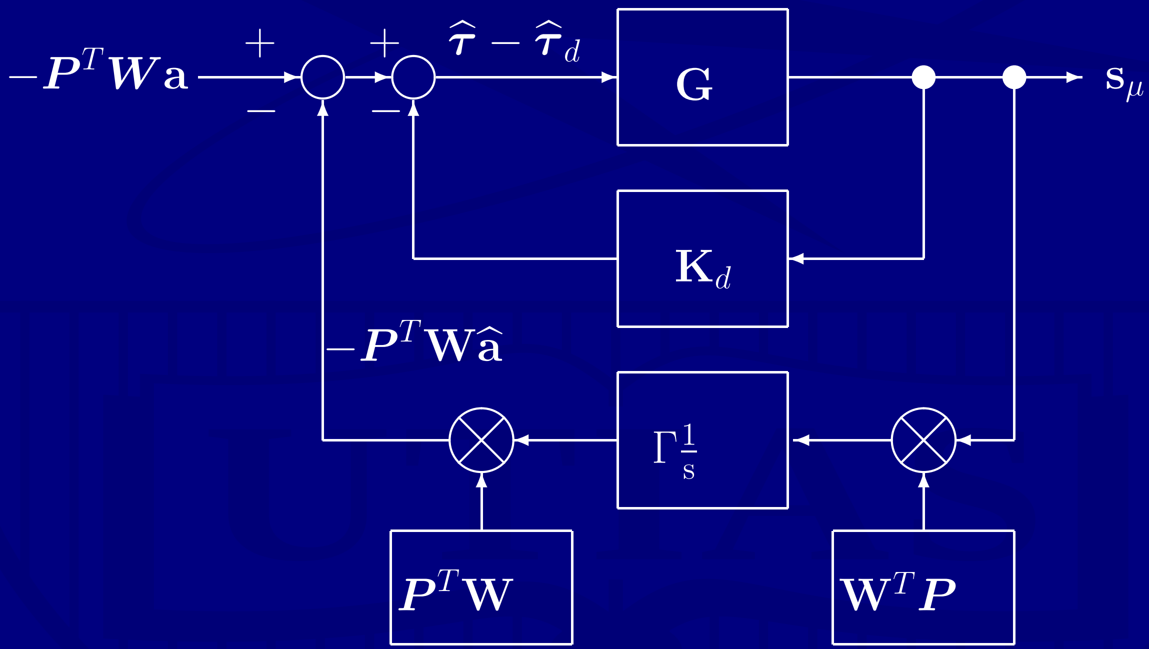
control law:

$$\begin{aligned}\widehat{\boldsymbol{\tau}} &= \boldsymbol{P}^T\boldsymbol{W}(\dot{\boldsymbol{\nu}}_r,\boldsymbol{\nu}_r,\boldsymbol{\nu})\widehat{\mathbf{a}}(t)-\mathbf{K}_d\mathbf{s}_\mu \\ &= \boldsymbol{P}^T[\widehat{\boldsymbol{M}}\dot{\boldsymbol{\nu}}_r+\boldsymbol{\nu}_r^{\otimes}\widehat{\boldsymbol{M}}\boldsymbol{\nu}]-\mathbf{K}_d[\dot{\widetilde{\boldsymbol{\rho}}}_\mu+\Lambda\widetilde{\boldsymbol{\rho}}_\mu]\end{aligned}$$

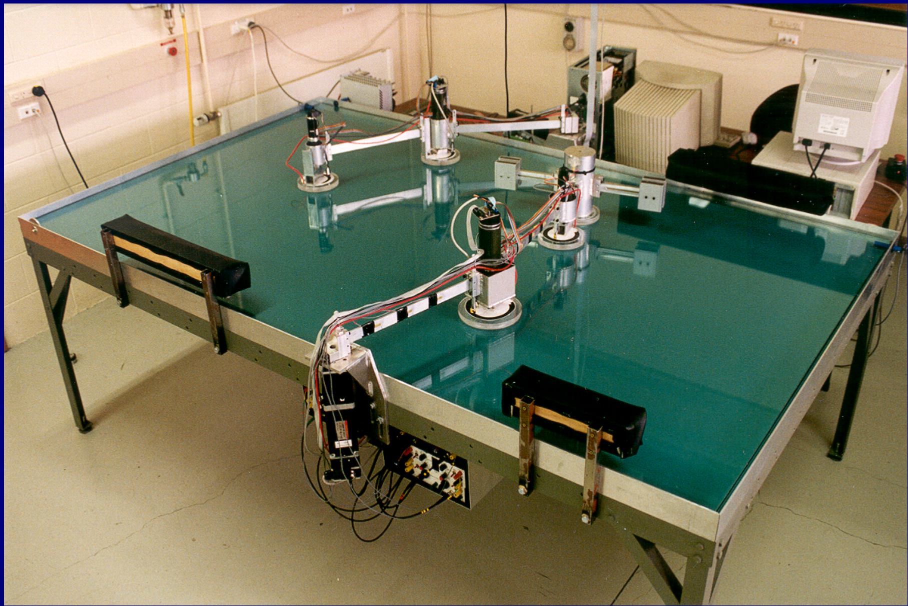
adaptation law:

$$\begin{aligned}\dot{\widehat{\mathbf{a}}} &= -\Gamma\boldsymbol{W}^T(\dot{\boldsymbol{\nu}}_r,\boldsymbol{\nu}_r,\boldsymbol{\nu})\boldsymbol{P}(\boldsymbol{\rho})\mathbf{s}_\mu, \\ &\quad \Gamma=\Gamma^T>\mathbf{O}\end{aligned}$$

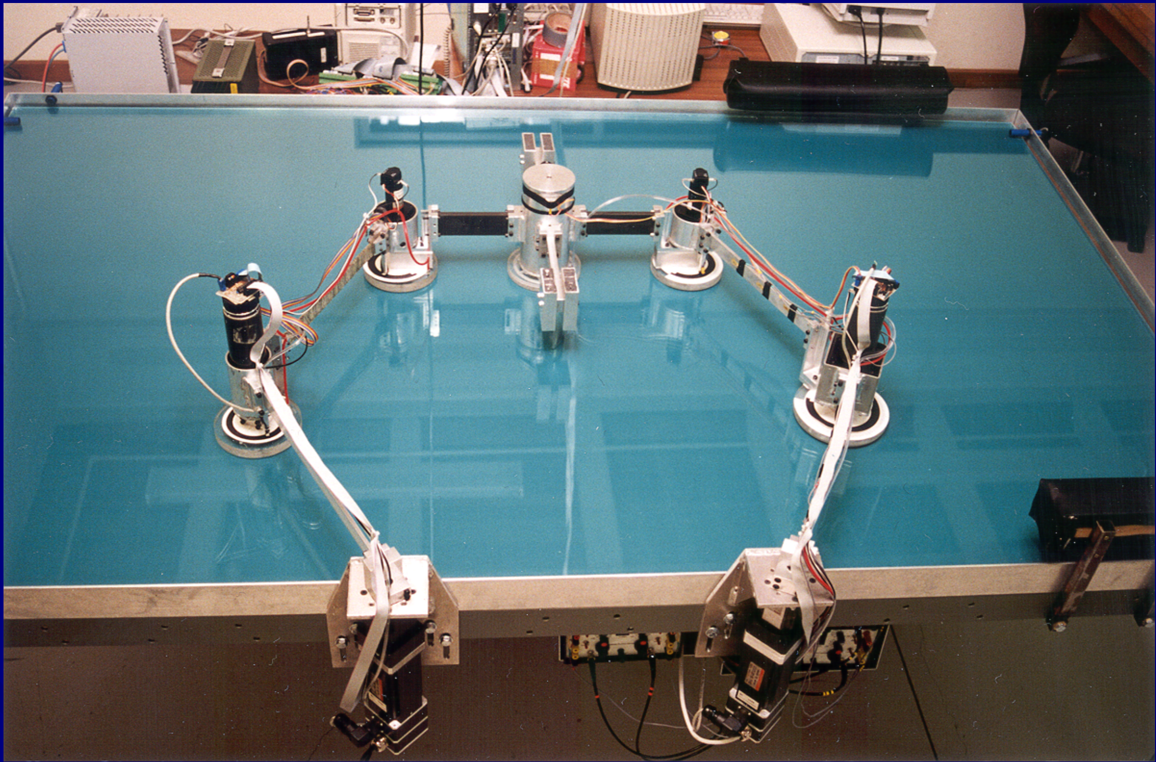
# The Adaptive Controller II



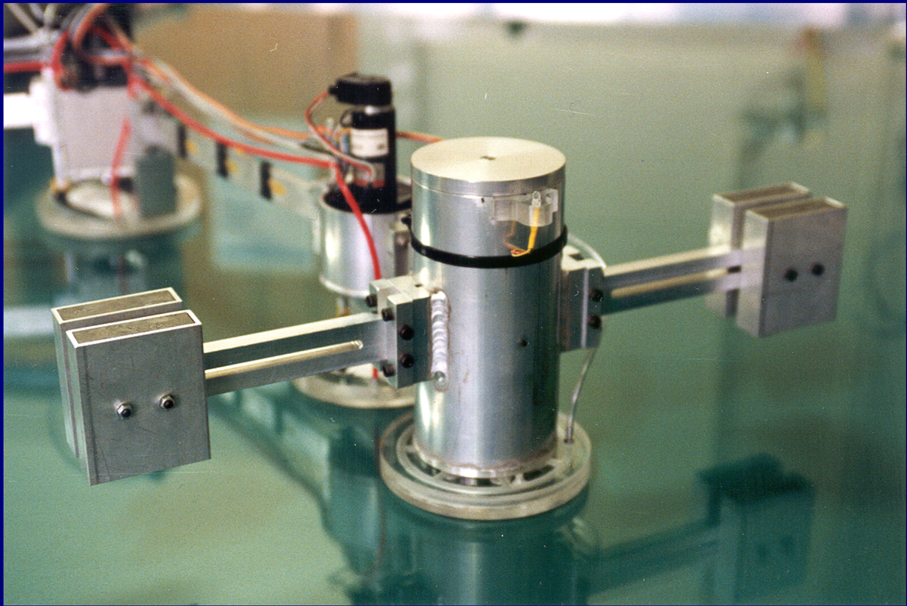
# Experimental Apparatus



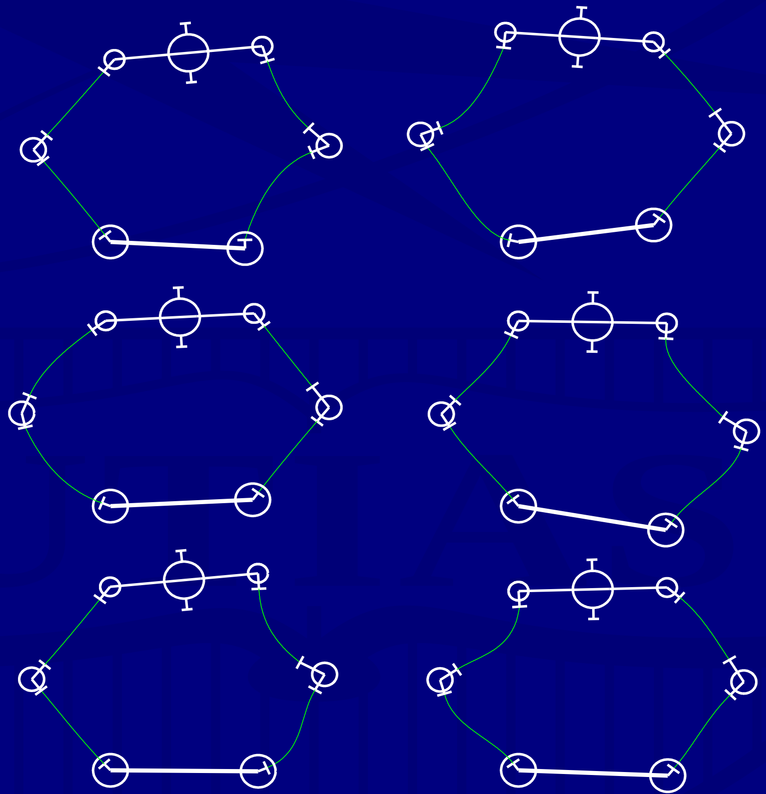
# Closed-Loop Configuration



# Payload

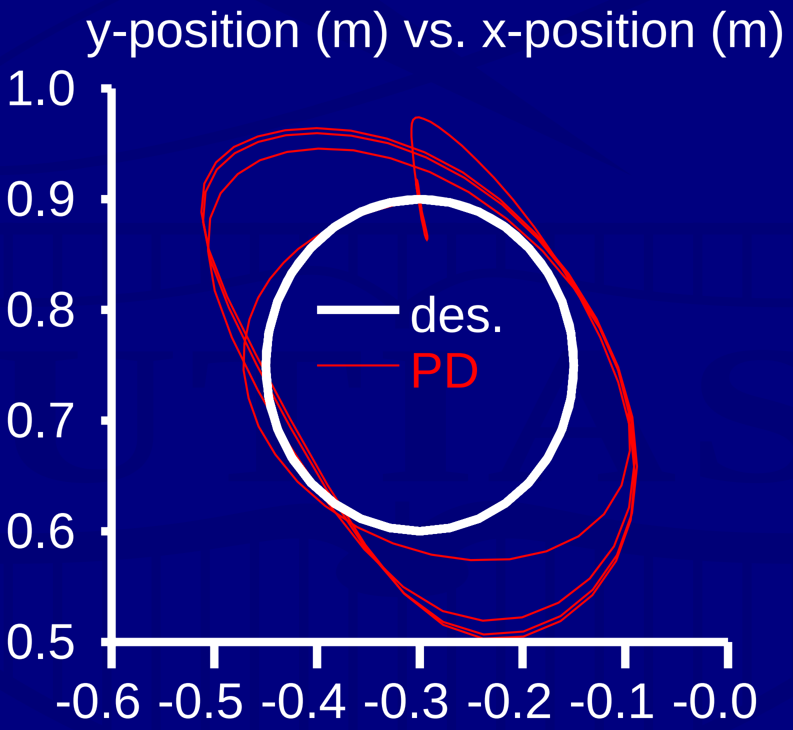


# Mode Shapes

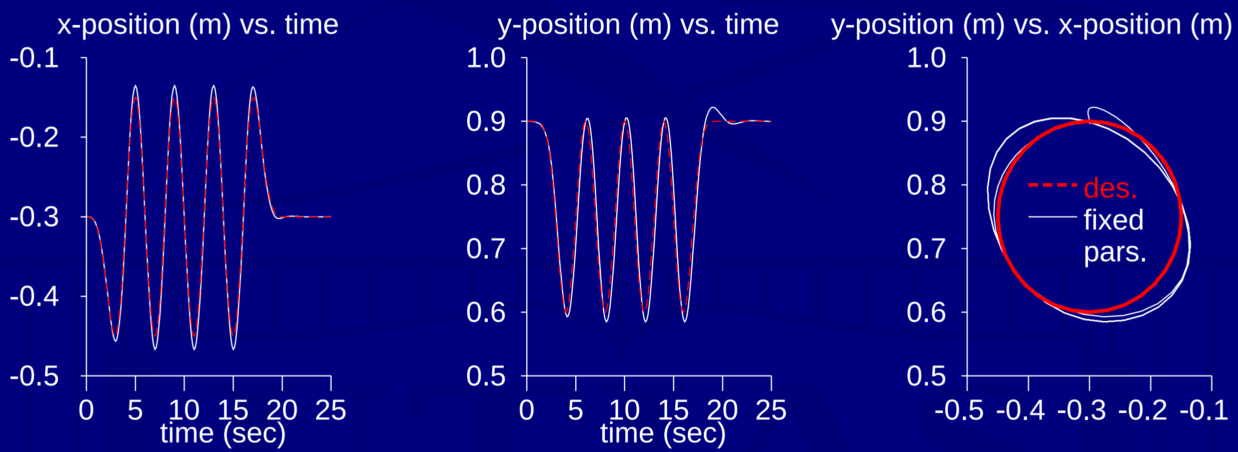


# PD Feedback Alone $(C_1 = C_2 = 0.5,\ \mu = 0.8)$



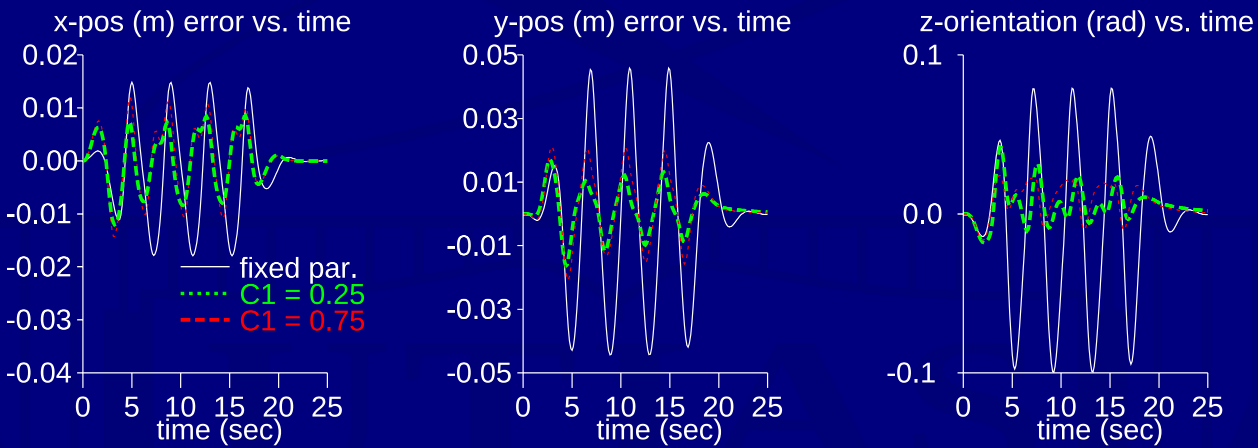


# Nonadaptive Results $(C_1 = C_2 = 0.5)$

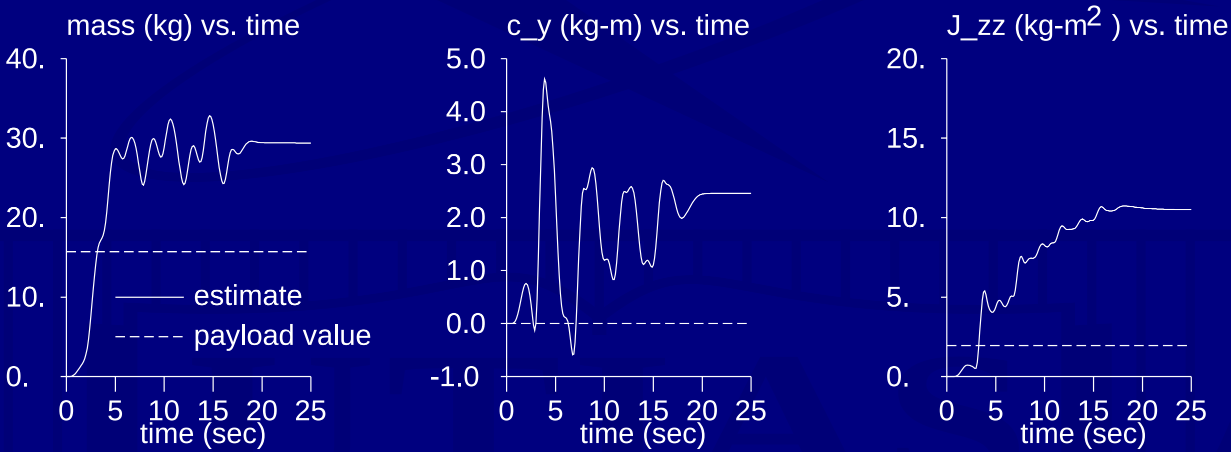


# Adaptive Results





# Parameter Estimates



# Summary of Presentation



- Passivity-based adaptive control: $\mu$-tip rates + load-sharing
- Adaptive feedforward depends only on “payload equations”
- Robust since passivity depends only on a large payload
- Results exhibit good tracking