Jumping Cat Robot with kicking a Wall

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Abstract

In this paper we study a robotic system moving in a vertical direction mimicking a cat's behavior as a cat kicks a wall to jump up to a roof, which may be an efficient mechanism for vertical movement and the robot system can be considered as one of the prototypes for Super Mechano Systems (SMS). Concept, modeling, controller design, simulation and experimental results are discussed.

Figure 1: Motion of the jumping cat

1. Introduction

Cats sometimes jump toward a wall and kick it to get to a higher-place like a roof (Fig.1), and can move in a vertical direction as a result, and the motion seems to be very skillful and efficient. Considering the movement from a viewpoint of constraints, a robotic system, which realizes the motion to change its configuration according to the position (on the ground, kicking with one leg, in the air, etc), can be considered as one of the prototypes for Super-Mechano Systems.

Our purpose in this paper is to analyze and construct a control law for a real machine mimicking the cat's motion. In the considered robotic system, the robotic motion is assumed to be constrained in the sagittal plane to make the problem simple.

In section 2, the dynamic equation of the system is derived, and in the following section a control method will be discussed. In section 4, we will show some simulation results. Finally we will show some experimental results and future work will be discussed.

2. Modeling

2.1. System structure

Even though a real cat twists its body after jumping to get to a higher place(the roof), we restrict the jumping-motion in the sagittal plane in this paper so that we can analyze and consider fundamental control problems.

In order to realize the motions, jumping from the ground to a wall and from the wall to the roof, we consider a 7-link robot as in Fig.2 The feature of the system can be summarized as follows.

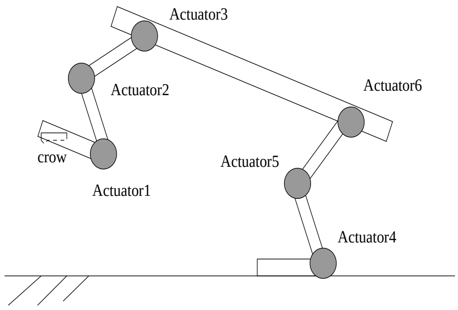

Figure 2: Robotic system

The robot has

- 6 rotational actuators at it's joints,
- when the toe makes contact with the floor or the wall, it is assumed that there is no slip between the toe and the contact point along the surfaces.

2.2. Modeling

The whole system is divided into two 4-link serial links at point X(Fig 3) virtually, and a holonomic body constraint $C_b(q) = 0$ to keep the body as one straight link is introduced.

As generalized coordinate systems, we use

$$q_F^T = [x_F, y_F, \theta_{F_1}, \theta_{F_2}, \theta_{F_3}, \theta_{F_4}], \quad (1)$$
$$q_R^T = [x_R, y_R, \theta_{R_1}, \theta_{R_2}, \theta_{R_3}, \theta_{R_4}], \quad (2)$$
$$q^T = [q_F^T, q_R^T]. \quad (3)$$

Though these coordinate systems are redundant and the system description becomes complex, the advantages are as follows:

- A 4-link dynamic equation is simpler than a 7-link one and we can use the same equation for each link,
- this coordinate system is very useful for judging the timing of switching the constraint on the toe, which will be mentioned later.

For the following discussion, Jacobian of the body constraint is defined as follows:

$$\frac{d}{dt}C_b(q) = J_b(q)\dot{q} = 0, \;\; J_b(q) = \frac{\partial C_b(q)}{\partial q}. \quad (4)$$

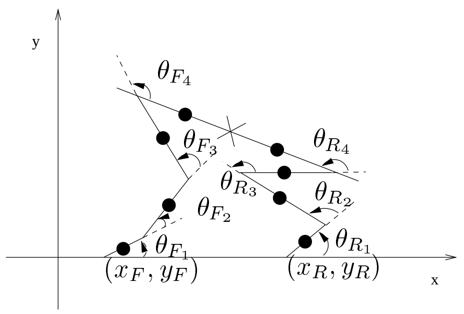

Figure 3: Robotic system and coordination

2.3. Variable constraints

We assume that enough constraint force is exerted when the toe makes contact with the ground or wall, and holonomic constraints $C_v(q, mode) = 0$ are introduced according to the state of the system, where mode is an index that indicates the state of the toe's contact.

For example, when only the hind toe is constrained to the floor or wall, $C_v(q, mode)$ becomes

$$C_v(q, mode) = \begin{bmatrix} x_R - X_R const \\ y_R - Y_R const \end{bmatrix}, \quad (5)$$

and we can calculate the Jacobian,as

$$\frac{d}{dt}C_v(q) = J_v(q)\dot{q} = 0, \quad J_v(q) = \frac{\partial C_v(q)}{\partial q}. \quad (6)$$

The problem here is how we can judge the mode. The answer lies in the understanding of the constraint force λ_v (Fig 4). In the case of toe being on the ground, λ_x is a horizontal constraint force and λ_y is vertical one, and if the λ_y is equal to zero and the acceleration upward is positive, the constraint should vanish($\lambda = 0$) and toe can move upward. When the toe is on the wall, switching timing depends on the λ_x vise versa.

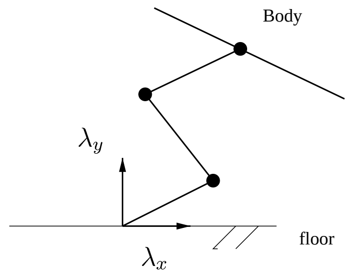

Figure 4: Constraint force

2.4. Dynamic equation

The dynamic equation for the system with the redundant coordinate systems is considered in this section. Ignoring the constraint, two 4-link manipulator's dynamic equations are described as

$$M(q)\ddot{q} + C(\dot{q}, q)\dot{q} + G(q) = \tau, \quad (7)$$

where

$$M = \begin{bmatrix} M_F & 0 \\ 0 & M_R \end{bmatrix}, \;\; C = \begin{bmatrix} C_F & 0 \\ 0 & C_R \end{bmatrix},$$
$$G = \begin{bmatrix} G_F \\ G_R \end{bmatrix}, \tau = \begin{bmatrix} \tau_F \\ \tau_R \end{bmatrix}.$$

In order to change position constraint to acceleration constraint, we differentiate (4) and (6), and in order to keep the constraint, constraint forces, $J_b^T \lambda_b$, and $J_v^T \lambda_v$ are introduced, and following simultaneous equations are used to express the system including all constraints:

$$M(q)\ddot{q} + C(\dot{q}, q)\dot{q} + G(q) = \tau - J_b^T \lambda_b - J_v^T \lambda_v, \quad (8)$$

$$J_b \ddot{q} = -\dot{J}_b \dot{q}, \quad (9)$$

$$J_v \ddot{q} = -\dot{J}_v \dot{q}. \quad (10)$$

From these equations, accelerating vector $\ddot{q}$, and constraint forces λ_b and λ_v can be calculated. Therefore, constraint force and acceleration can be used for judging the change of the mode.

Collision with the wall or other things is assumed to be perfectly inelastic, and the state will shift to that of the under constraint just after the collision which is modeled as effects of impulse forces.

3. Controller Design

Since the initial configuration is very important for the robot's jumping motion, it is determined by *stochastic dynamic manipulability measure*. For dynamic control of the robot's jump, we pay attention to the motion of the center of mass mainly, and the proposed method is derived as if the center of the gravity is moved by a spring connected to a virtual wall.

3.1. Stochastic dynamic manipulability measure

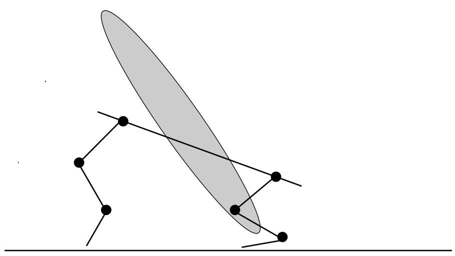

Figure 5: Realizable acceleration for the center of mass

There exists a lot of possible postures of the 7-link robot before the jump, and it is not easy to determine what kind of pose is suitable for the jump. Therefore, we use a measure to decide the position for jumping, i.e., *stochastic dynamic manipulability measure*, which evaluates an expected required torque to realize the desired acceleration.

Sub-optimal configuration for the jump from the ground and from the wall based on the stochastic dynamic manipulability measure can be determined by a numerical optimization.

In another words, at first we determine the pose which easily achieves the desired acceleration and angular acceleration, and the robot is set to the posture before the jump.

Let's assume that, (x_g, y_g) and θ_b indicate the coordinates of the center of mass, and angle of the body, respectively. By eliminating the constraint force λ from the system dynamics, we have

$$\tau_a = M_a \ddot{q} + C_a \dot{q} + G_a, \tag{11}$$

where

$$\dot{x_c} = J_q \dot{q}, \qquad x_c = \begin{bmatrix} x_g & y_g & \theta_b \end{bmatrix}^T, \tag{12}$$

and

$$\begin{aligned} M_a &\equiv M, \\ C_a &\equiv YC + J_r^T X^{-1} \dot{J}_r, \\ G_a &\equiv YG, \\ \tau_a &\equiv Y\tau, \\ X &\equiv J_r M^{-1} J_r^T, \\ Y &\equiv I - J_r^T M^{-1} J_r M^{-1}. \end{aligned}$$

For this description of the system, stochastic dynamic manipulability measure is defined as

$$w_{sd} = \begin{cases} \sqrt{\dfrac{tr[W^T W]}{tr[W^T\{(J_q M^{-1})(J_q M \ -1)\}^{-1} W]}} \\ \qquad\qquad (\det[J_q J_q^T] \neq 0) \\ 0 \\ \qquad\qquad (\det[J_q J_q^T] = 0), \end{cases} \tag{13}$$

where

w_{sd}: Stochastic dynamic manipulability measure

W: Weight matrix indicates the direction to accelerate

J_q: Jacobian matrix.

To determine the sub-optimal configuration with respect to the measure under the constraints, w_{sd} is updated by the following iteration:

$$\begin{aligned} w_{sd}(i+1) &= w_{sd}(i) + \left(\frac{\partial w_{sd}(i)}{\partial q}\right)^T \Delta q, && (14) \\ \Delta q &= Dq\epsilon, && (15) \\ \epsilon &= \{(\frac{\partial w_{sd}(i)}{\partial q})^T Dq\}^T \cdot k, && (16) \end{aligned}$$

where Dq is basis of $Ker(J_q)$. If we use the iteration and k is a semi-positive constant, W_{sd} is increased as

$$w_{sd}(i+1) = w_{sd}(i) + \|(\frac{\partial w_{sd}(i)}{\partial q})^T Dq\|^2 k \geq w_{sd}(i). \tag{17}$$

3.2. Jumping control

Only jumping with both legs or with only the hind leg is mentioned here. When the cat is in the air, we just adopt feedback control so that the posture of the legs converges to a desired one determined beforehand by the above method, which is ready for the next jump.

In order to derive a proposed control algorithm, we re-describe the system, and we pay attention to the center of mass and the body's angle. Using a coordinate change from q to $x_c = [x_g, y_g, \theta_b]$, the system can be expressed as

$$\ddot{x}_c = A(q, \dot{q})\tau + B(q, \dot{q}), \quad \tau = [\tau_F^T, \tau_R^T]^T, \tag{18}$$

where

$$A \equiv J_q M_a^{-1} Y,$$
$$B \equiv J_q M_a^{-1}(C_a + G_a) - \dot{J}_q \dot{q},$$
$$Y \equiv I - J_r^T M^{-1} J_r M^{-1},$$

and J_q is the tangential map of the coordinate transformation. For the system representation, $\ddot{x}_c$ is determined so that the motion of the mass center follows a simple mass-spring model.

3.2.1. Jumping with both legs

PSfrag replacements

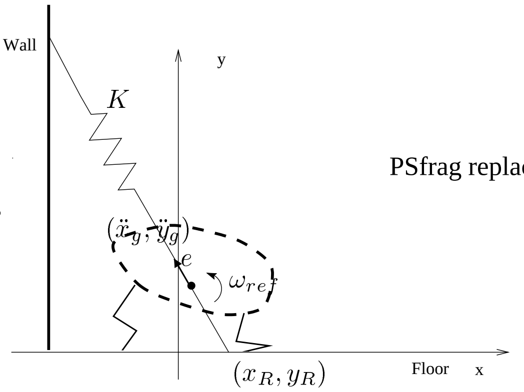

Figure 6: Model matching 1

ω_{ref}:	Desired angular acceleration
θ_{ref}:	Desired angle
e :	Normal unit directional vector
K :	Coefficient of the virtual spring
l :	Length of the virtual spring
M :	Mass of the virtual body

The desired acceleration of the body $\ddot{x}_{ref1} \in R^2$ is, as shown in Fig.6, determined so that the virtual mass concentrated to the mass center moves as if it is pulled by a strong spring stuck to the wall, and a desired angular acceleration of body $\ddot{x}_{ref2} \in R$ is designed to rotate in the desired direction as

$$\ddot{x}_{ref} = \begin{bmatrix} \ddot{x}_{ref1} \\ \ddot{x}_{ref2} \end{bmatrix} \quad (19)$$

$$= \begin{bmatrix} \frac{lK}{M} e \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \dot{\omega}_{ref} - K_\theta(\dot{\theta}_b - \omega_{ref}) \end{bmatrix}, \quad (20)$$

where e is a unit vector toward the desired direction for jumping, and v is a vector from the center of mass to the wall along to the line from the toe to the center of mass, and $l =< e, v >$ indicates the spring's length. K_θ is an appropriate feedback gain.

Because of the redundant system, τ is selected to minimize the following criterion function as:

$$J_1 := \|W_c(A\tau - B - \ddot{x}_{ref})\|^2 + \frac{1}{2}\tau^T W_\tau \tau, \quad (21)$$

where

$$W_c = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \beta \end{bmatrix}, \quad (22)$$

and α, β are positive constants, and W_τ is an appropriate positive weight matrix.

3.2.2. Jumping only with the hind leg

PSfrag replacements

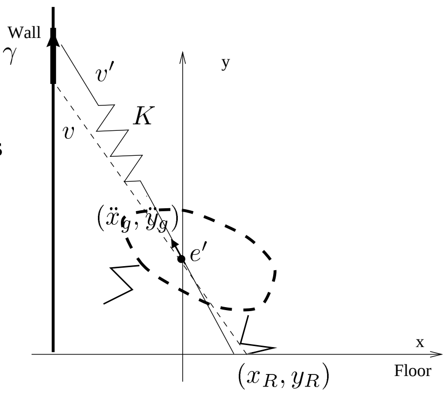

Figure 7: Model matching 2

v :	Normal directional vector of the spring
v' :	Modified unit vector for desired direction
γ :	Modifying ratio of the direction
e' :	Normal unit directional vector
K:	Coefficient of the virtual spring
l :	Length of the virtual spring
M:	Mass of the virtual body

After the motion with the two legs, the front leg will naturally lift. In order to control the center of mass, only the hind leg's actuators are mainly used, and as it is difficult to control all of the three degree of freedom,the spring-mass model is modified to control the body angle indirectly First, v is determined as a directional vector by connecting the toe and the center of mass, then, it is adjusted to a directional vector v' due to the sign of the error of the angular velocity by γ as follows:

$$v' = v - [0, \gamma sgn(\omega_{ref} - \dot{\theta}_b)]^T \quad (23)$$

and, desired acceleration x_{ref} is determined by

$$\ddot{x}_{ref1} = \begin{bmatrix} \frac{lK}{M} e' \\ 0 \end{bmatrix}, \quad (24)$$

$$l := < v, e' >, \quad e' := v'/\|v'\|,$$

and the weight W_a is also changed to

$$W_a = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{25}$$

As in the previous method, τ_R is determined to minimize the following criterion function:

$$J_2 := \|W_a(A\tau - B - \ddot{x}_{ref})\|^2 + \frac{1}{2}\tau^T W_\tau \tau, \tag{26}$$

where τ_L is determined locally.

4. Simulation

In order to examine the validity of the proposed method, we conducted numerical simulations of the jump for a model of an experimental system. In the optimization and the simulations,it is assumed that the fore leg and hind sub-systems have the same parameters.

4.1. Pose optimization

We obtained a sub-optimal posture, using the stochastic dynamic manipulability measure from some of the initial poses by a gradient search. One of the results is shown in Fig.8 and Fig.9. By the procedure, the manipulability measure increased by about 20%, from $s_{sd} = 37.6$ to $s_{sd} = 43.5$.

4.2. Cat jumping with kicking a wall

In the simulation, we assumed that a wall is located at $x = -0.4$[m], and the roof at $y = 0.5$[m].

For the values of control parameters, we used $K = 700$[N/m], $\omega = -40[rad/s]$, and when the cat jumps from the floor to the wall, we set $\omega = -800[rad/s]$, from the wall to the roof. Since there is no systematic rule to determine the value of parameters, they are determined by trial and error.

As shown in Fig.10 the sequence of the jumping motion is shown. It is shown that the robotic cat jumped up to a roof at y=0.5[m] after kicking the wall with a rotation.

5. Experimental Results

In order to check the validity of the proposed method, we constructed an experimental system.

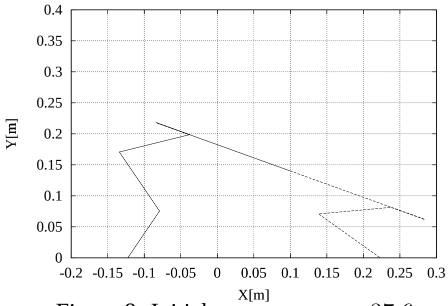

Figure 8: Initial posture: $w_{sd} = 37.6$

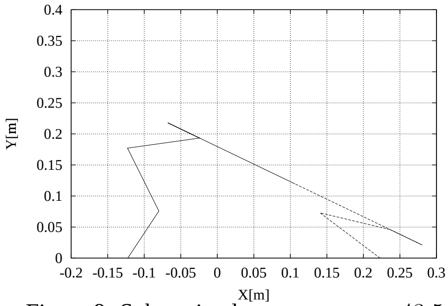

Figure 9: Sub-optimal posture: $w_{sd} = 43.5$

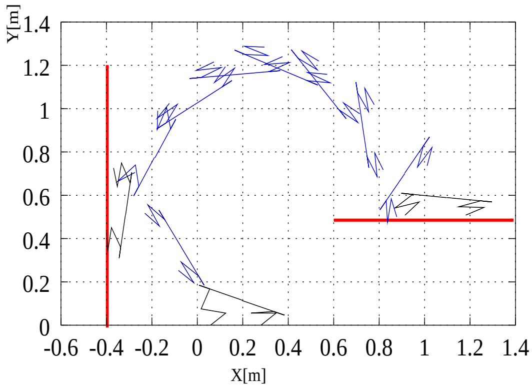

Figure 10: Simulation result: Jump kicking the wall

5.1. System configuration

We designed a 7-link robotic cat shown in Fig.11. The system configuration of the whole system is given in Fig.12. The following describes the details of the robotic system.

Since motors are too heavy to be installed in the robot, power of the actuators is supplied from outside by wires (Fig.13), and the weight of the wires is compensated for by a counter weight.

The position and the angle of the toe is measured by the CCD camera. In order to measure the angles of the joints, potentiometers are used since encoders of the motor are useless due to the extension of the

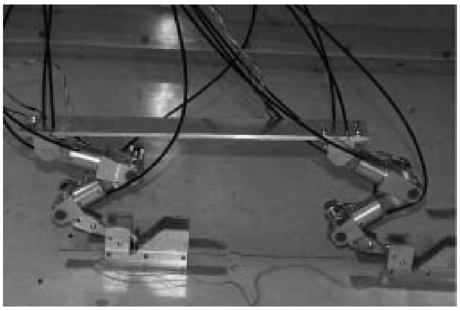

Figure 11: Cat robot

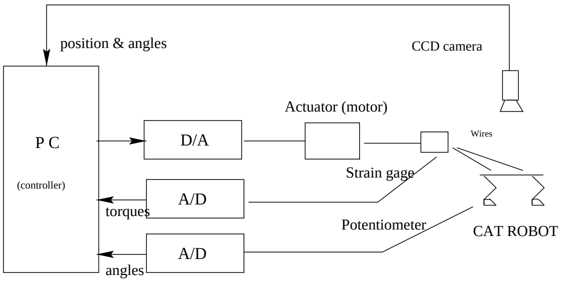

Figure 12: System configuration

wire. Because of the wires extension, there are a lot of time delays in the wire system. In order to control the angle in such a bad condition, sliding mode control are introduced as,

$$\tau_d = -K_{\text{outer}} sgn(S_1), \quad (27)$$

$$S_1 = \dot{e}_q + \lambda_{\text{outer}} e_q, \quad (28)$$

where $e_q = q - q_d$ and λ is a positive constant.

Furthermore, we introduced strain-gages in the wire system between motors and the body of the cat to measure the equivalent torques exerted to the joints(Fig.15), since the power applied by the motors is lost due to friction in the wire system. Tension of the wire is measured using the bending deformation of a plate stretched from both sides(Fig.16).

Using the information from the strain gage, a minor loop compensation is constructed as,

$$\tau = \tau_d - K_{minor} sgn(S_2), \quad (29)$$

$$S_2 = \int_0^t e_\tau dt, \quad (30)$$

where, $e_\tau = \tilde{\tau} - \tau_d$, $\tilde{\tau}$ is the measured torque.

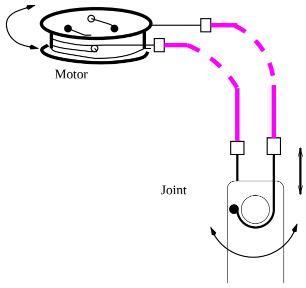

Figure 13: Motor and wire system

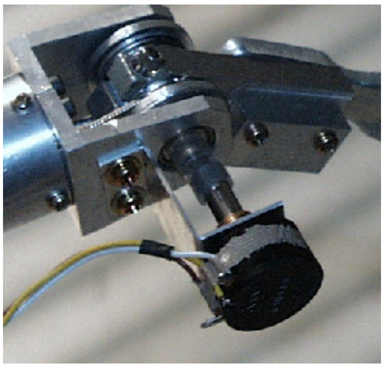

Figure 14: Potentiometer

5.2. Learning control

The problem due to the time loss and disturbance of the wire system is too serious, it was difficult to apply the proposed method directly, therefore the learning control [6] is applied as follows.

In the i-th trial, the sliding surface is modified as

$$S_1(t) = \dot{e}_q(t) + \lambda e_q(t) + u_i(t), \quad (31)$$

where u_i indicates the learning term which is updated by the following algorithm.

$$u_{i+1} = u_i - \gamma_i L(q_d - q_i), \quad (32)$$

where, L is a learning filter, q_i is the experimental data of the ith trial, and γ_i is positive coefficient. The desired angle q_d is determined based on the simulation data. (See the details in [6].)

One of the results is shown in the Fig.17. It is observed that the output of the latter trial is obviously improved.

6. Conclusion and Future Work

We proposed a jumping method of a robotic cat using spring-mass model matching, and we conformed the

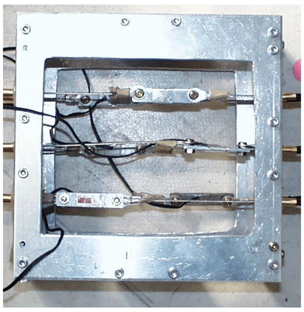

Figure 15: Strain gage system

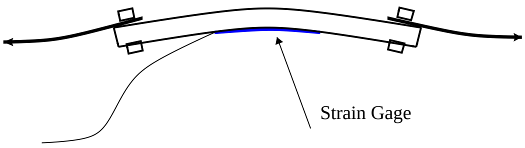

Figure 16: Principal of measuring torque

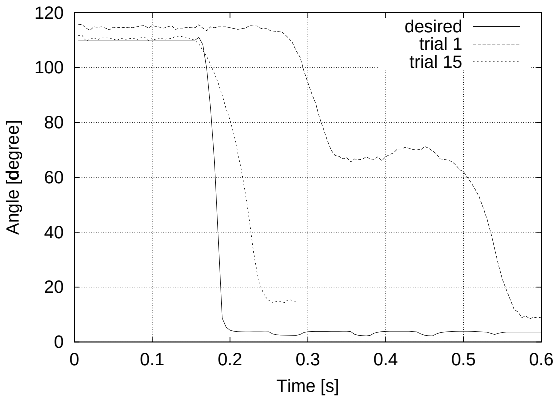

Figure 17: Learning result: Angle of the rear toe

effectiveness of the control law by simulation, in which the robotic cat could jump towards the wall and land on the wall. For it's initial posture evaluation, we used *stochastic dynamic manipulability measure*.

In order to realize the real robot, VSS control and learning control were applied and much progress are conformed. But the jumping has not completed yet, and future work should include:

- completion of the jumping with the real robot,
- learning control in the task space,
- realization of a 3D cat robot, and development of the control into the three-dimensional system.

Acknowledgment
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