Unit Design of Hyper-redundant Snake Robots Based on a Kinematic Model

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Abstract

We define the redundancy controllable system of hyper-redundant mechanical systems. We derive the condition that the hyper-redundant snake robots become redundancy controllable, and the control law with considering the redundancy. We also propose a concept of a unit and the system design strategy of the snake robots. Simulation results are shown.

1. Introduction

Unique and interesting gait of the snakes makes them able to crawl, climb a hill, climb a tree by winding and move on very slippery floor [1]. It is useful to consider and understand the mechanism of the gait of the snakes for mechanical design and control law of snake robots.

Hirose has long investigated snake robots and produced several snake robots, and he models the snake by a wheeled link mechanism with no side slip [2]. Some other snake-like mechanisms are developed in [3] and [4]. Burdick and Chirikijian discuss the sidewinding locomotion of the snake robots based on the kinematic model [5]. Ostrowski and Burdick analyze the controllability of a class of nonholonomic systems that the snake robots are included on the basis of the gemotric approach [6]. The feedback control law for the snake head's position using Lyapunov method has been developed by Prautesch et al. on the basis of the wheeled link model [7]. They point out the controller can stabilize the head position of the snake robot to its desired value, but the configuration of it converges to a singular configuration. From the model we find that the snake robot does not have the redundant degrees of freedom, and this leads to the difficulty in the control objective of the singular configuration avoidance.

In this paper we define the redundancy controllable system and propose control law and structure design methodology of redundant snake robots based on the wheeled link model. We find that introduction of links without wheels and shape controllable points in the snake robot's body makes the system redundancy controllable. In this case the head's velocity of the snake robot does not determine all joint velocities of the robot uniquely. We introduce the cost function related to the measure for the singularity and the manipulability of the system, and construct a controller with considering the redundancy. Using redundancy, it becomes possible to accomplish both the main objective of controlling the position and the posture of the snake robot head and the shape of the snake robot, and the sub-objective of the singular configuration avoidance and the obstacle avoidance.

We introduce an unit which is fundamental element of the snake robots. We assume that the serial connection of uniform units constructs a snake robot. We discuss the condition of the unit that the connected system becomes redundancy controllable. We propose the unit design and the connection law for constructing the snake robot.

From simulation results we find that the crawling motion of the snake robot is natural.

2. Redundancy Controllable System

Let $\boldsymbol{q} \in R^{\bar{n}}$ be generalized coordinates, $\boldsymbol{u} \in R^{\bar{p}}$ be the input vector, $\boldsymbol{w} \equiv S\boldsymbol{q} \in R^{\bar{q}}$ be the state vector to be controlled, S be a selection matrix, whose row vectors are independent unit vectors, related to generalized coordinates. We define that the system

$$A(\boldsymbol{q})\dot{\boldsymbol{w}} = B(\boldsymbol{q})\boldsymbol{u} \quad (1)$$

is redundancy controllable if the number of inputs $\bar{p}$ is greater than that of the state vector to be controlled $\bar{q}$ ($\bar{p} > \bar{q}$), the matrix A is full column rank, B is full row rank, and there exists an input $\boldsymbol{u}$ which accomplishes both the main objective of the convergence of the vector $\boldsymbol{w}$ to the desired state $\boldsymbol{w}_d$ ($\boldsymbol{w} \to \boldsymbol{w}_d, \dot{\boldsymbol{w}} \to \dot{\boldsymbol{w}}_d$) and the sub-objective of increase (or decrease) of a cost function $V(\boldsymbol{q})$.

For a snake robot based on the wheeled link model we discuss a condition that the system is redundancy controllable.

3. Kinematic Model of Hyper-redundant Snake Robots

We consider a redundant n-link snake robot. Let n be the number of links, m be the number of wheeled links, $[\,x_h \quad y_h \quad \theta_h\,]^T$ be the vector of the position and the posture of the snake head, $[\,\phi_1 \quad \cdots \quad \phi_{n-1}\,]^T$ be the vector of relative joint angles and $\boldsymbol{q} = [\,x_h \quad y_h \quad \theta_h \quad \phi_1 \quad \cdots \quad \phi_{n-1}\,]^T \in R^{n+2}$ be the generalized coordinates.

The length of each link is $2l$. The wheels are located on the middle point of the wheeled link. Let $[\,x_i \quad y_i\,]^T$ be the position vector of the middle point of the link i as shown in Fig. 1. As the wheel does not slip to the side direction, the velocity constraint condition should be satisfied. If the i-th link is wheeled, the constraint can be written as

$$\dot{x}_i \sin(\theta_h + \sum_{k=1}^{i-1} \phi_k) - \dot{y}_i \cos(\theta_h + \sum_{k=1}^{i-1} \phi_k) = 0. \quad (2)$$

From the geometric relation the position vector is expressed as

$$x_i = x_h + 2l\cos\theta_h + 2l\sum_{k=1}^{i-2}\cos(\theta_h + \sum_{j=1}^{k}\phi_j) + l\cos(\theta_h + \sum_{k=1}^{i-1}\phi_k) \quad (3)$$

$$y_i = y_h + 2l\sin\theta_h + 2l\sum_{k=1}^{i-2}\sin(\theta_h + \sum_{j=1}^{k}\phi_j) + l\sin(\theta_h + \sum_{k=1}^{i-1}\phi_k). \quad (4)$$

Substituting (3), (4) into (2), gives the velocity constraint equation

$$A(\boldsymbol{q})\dot{\boldsymbol{w}} = B(\boldsymbol{q})\boldsymbol{u}, \quad \boldsymbol{u} = \dot{\boldsymbol{\theta}} \quad (5)$$

where $\boldsymbol{w}$ is the state vector to be controlled, $\boldsymbol{\theta}$ is the vector of the active joint angles, $A \in R^{m \times q}, B \in R^{m \times p}$, and the angular velocity of the active joint is regarded as the input of the system.

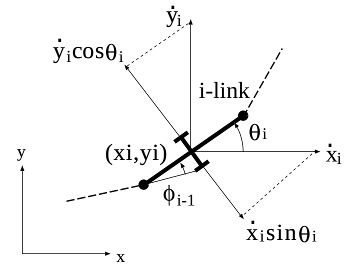


Fig. 1 Velocity constraint of the i-th wheeled link

As one wheeled link has one velocity constraint, the number m of the wheeled links is equal to the number of equations. We assume that at least the snake head's position and posture are controlled.

4. Condition for Redundancy Controllable System

We consider an n-link snake robot whose all links are wheeled as shown in Fig. 2. Let $\bar{\boldsymbol{w}} = [\,x_h \quad y_h \quad \theta_h\,]^T$ be the position and posture of the snake head, $\bar{\boldsymbol{\theta}} = [\,\phi_1 \quad \cdots \quad \phi_{n-1}\,]^T$ be relative angles of each link, $\boldsymbol{q} = [\,\bar{\boldsymbol{w}}^T \quad \bar{\boldsymbol{\theta}}^T\,]^T$ be generalized coordinates. We assume the angular velocity of an active joint is regarded as an input of the system

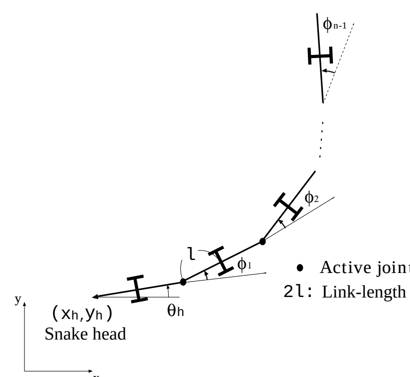


Fig. 2 An n-link snake robot

As in this case all links are wheeled link, the system can be written as

$$\bar{A}(\boldsymbol{q})\dot{\bar{\boldsymbol{w}}} = \bar{B}(\bar{\boldsymbol{\theta}})\bar{\boldsymbol{u}}, \quad \bar{\boldsymbol{u}} = \dot{\bar{\boldsymbol{\theta}}} \quad (6)$$

where $\bar{A} \in R^{n \times 3}, \bar{B} \in R^{n \times (n-1)}$. In the system (6), as the velocity constraint of the passive wheel of the head-link is expressed as $\dot{x}_h \sin\theta_h -$

$\dot{y}_h \cos\theta_h - l\dot{\theta}_h = 0$, we find that the matrix $\bar{B}$ is not full row rank. It is necessary that the first link is wheel free.

Next, we consider the system that only the head-link is wheel free. This system can be written as

$$\bar{A}^{'}(\boldsymbol{q})\dot{\bar{\boldsymbol{w}}} = \bar{B}^{'}(\bar{\boldsymbol{\theta}})\bar{\boldsymbol{u}} \tag{7}$$

where $\bar{A}^{'} \in R^{(n-1)\times 3}, \bar{B}^{'} \in R^{(n-1)\times(n-1)}$ and

$$\bar{B}^{'} = \begin{bmatrix} l & & & & \\ b_{21} & l & & O & \\ \vdots & & \ddots & & \\ \vdots & & & l & \\ b_{(n-1)1} & \cdots & \cdots & b_{(n-1)(n-2)} & l \end{bmatrix}.$$

We find that the matrix $\bar{B}^{'}$ is invertible. As the velocity of the snake head $\dot{\bar{\boldsymbol{w}}}$ determines the input $\bar{\boldsymbol{u}}$ uniquely, the system (7) is not redundant.

In this paper, as we control the shape of the snake robot body in addition to the position and posture of the snake head, some relative angles are included in the state vector to be controlled. We call the joint angle the shape controllable point and the number of the shape controllable points the shape controllability index. Let s be the shape controllability index.

Next, we consider that ϕ_i is introduced as the controlled state. Then the system can be written as

$$\hat{A}(\boldsymbol{q})\dot{\hat{\boldsymbol{w}}} = \hat{B}(\bar{\boldsymbol{\theta}})\hat{\boldsymbol{u}} \tag{8}$$

where $\hat{\boldsymbol{w}} = [\,\bar{\boldsymbol{w}}^T \quad \phi_i\,]^T$, $\hat{\boldsymbol{u}} = [\,\dot{\phi}_1 \cdots \dot{\phi}_{i-1}\ \dot{\phi}_{i+1} \cdots \dot{\phi}_{n-1}\,]$, $\hat{A} \in R^{(n-1)\times 4}, \hat{B} \in R^{(n-1)\times(n-2)}$ and

$$\hat{B} = \begin{bmatrix} l & & & & \\ b_{21} & \ddots & & O & \\ \vdots & & & & \\ b_{(i-1)1} & \cdots & l & & \\ b_{i1} & \cdots & b_{i(i-1)} & & \\ b_{(i+1)1} & \cdots & b_{(i+1)(i-1)} & l & \\ \vdots & & & & \ddots \\ b_{(n-1)1} & \cdots & b_{(n-1)(i-1)} & b_{(n-1)(i+1)} \cdots & l \end{bmatrix}.$$

We find possibility that the matrix $\hat{B}$ is not full row rank. For necessity of the full row rankness of the B matrix, the link which is introduced the shape controllable point should be wheel free.

Next, we consider the system which the $i+1$ $,\cdots,$ the n-th links are wheel free. This system can be written as

$$\tilde{A}(\tilde{\boldsymbol{q}})\dot{\bar{\boldsymbol{w}}} = \tilde{B}(\bar{\boldsymbol{\theta}})\bar{\boldsymbol{u}} \tag{9}$$

where $\tilde{A} \in R^{(i-1)\times 3}, \tilde{B} \in R^{(i-1)\times(n-1)}$,

$$\tilde{B} = \left[\begin{array}{ccccc|c} l & & & & & \\ b_{21} & l & & O & & \\ \vdots & & \ddots & & & O \\ \vdots & & & l & & \\ b_{(i-1)1} & \cdots & \cdots & b_{(i-1)(i-2)} & l & \end{array}\right]$$

As the column vectors after the $(i+1)$-th column of the matrix $\tilde{B}$ are zero, we find that the movement of links behind the last wheeled link does not contribute to the movement of the snake robot head. To satisfy the condition (B is full row rank) so that the system is redundancy controllable, we should introduce the assumptions.

[assumption 1] : The head link is wheel free.

[assumption 2] : The tail link is the wheeled link.

[assumption 3] : The link which is introduced the shape controllable point is wheel free.

[assumption 4] : The passive joint angle is equivalent to the state variable to be controlled as the shape controllable point.

We remark that the joints of the wheeled links are passive/active joints and that the link which has the passive joint is wheel free. If the assumptions 1-4 are satisfied, the matrix B is full row rank. The assumptions 1-4 are the sufficient condition for the full row rankness of the matrix B.

We consider a redundant n-link snake robot as shown in Fig. 3 with m wheeled links which satisfies the assumptions 1-4. In this case the system can be written as

$$A(\boldsymbol{q})\dot{\boldsymbol{w}} = B(\boldsymbol{q})\boldsymbol{u} \tag{10}$$

where $A \in R^{m\times(3+s)}, B \in R^{m\times(n-1-s)}$, $\boldsymbol{w} = S\boldsymbol{q} \in R^{(3+s)}, \boldsymbol{\theta} = \bar{S}\boldsymbol{q} \in R^{(n-1-s)}$ and $\boldsymbol{u} = \dot{\boldsymbol{\theta}}$. We find that exclusion of $\boldsymbol{w}$ from $\boldsymbol{q}$ gives $\boldsymbol{\theta}$. $\{\boldsymbol{w}\} \cup \{\boldsymbol{\theta}\} = \{\boldsymbol{q}\}, \{\boldsymbol{w}\} \cap \{\boldsymbol{\theta}\} = \boldsymbol{\phi}$.

The necessary and sufficiant condition for the existance of the solution of the system (10) is

$$rank[A, B\boldsymbol{u}] = rank A. \tag{11}$$

In the case of $m < dim(\boldsymbol{w}) = q$ for the system (10), if an input $\boldsymbol{u}$ is given, then the solution $\dot{\boldsymbol{w}}$ does not determine uniquely. From the necessity of the uniqueness of the solution of the system (10), we introduce the condition $m \geq q = 3 + s$. We set $m < p$ so as to satisfy the condition that the velocity vector $\dot{\boldsymbol{w}}$ does not determine the control input $\boldsymbol{u}$ uniquely. This condition means the redundancy of the input.

From two conditions $m \geq q$ and $m < p$ we find that the condition $p > q$ is satisfied. The condition that the system (10) is redundancy controllable can be written as

$$\begin{cases} m & \geq & 3+s \\ m & < & (n-1)-s \end{cases}. \tag{12}$$

Combining them gives

$$3+s \;\leq\; m \;<\; (n-1)-s. \tag{13}$$

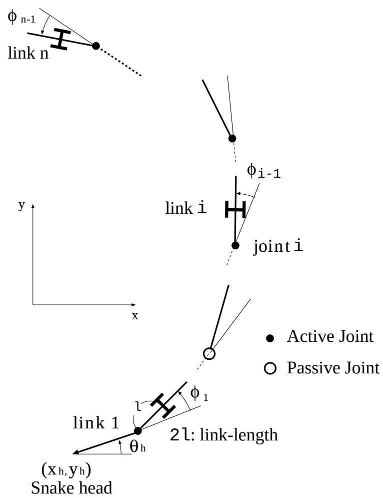


Fig. 3 A redundant n-link snake robot

5. Controller Design

Let us define the control input as follows:

$$\boldsymbol{u} = B^{*}A\{\dot{\boldsymbol{w}}_d - K(\boldsymbol{w}-\boldsymbol{w}_d)\} + (I_k - B^{*}B)\alpha\boldsymbol{\eta} \tag{14}$$

where B^{*} is a pseudo-inverse matrix of B, $\boldsymbol{\eta} = \nabla_{\boldsymbol{\theta}}V(\boldsymbol{q}) = [\,\partial V/\partial\theta_1 \cdots \partial V/\partial\theta_{n-1-s}\,]$ is the gradient of the cost function $V(\boldsymbol{q})$ with respect to the vector $\boldsymbol{\theta}$ related to the input vector $\boldsymbol{u}$, and $\alpha \geq 0$, $K > 0$. The first term of the right side of (14) is the control input term to accomplish the main objective of the convergence of the state vector $\boldsymbol{w}$ to the desired value $\boldsymbol{w}_d$. As the second term $(I - B^{*}B)\alpha\boldsymbol{\eta}$ belongs to the null space of the matrix B, we obtain

$$B\boldsymbol{u} = A\{\dot{\boldsymbol{w}}_d - K(\boldsymbol{w}-\boldsymbol{w}_d)\}. \tag{15}$$

As the vector $B\boldsymbol{u}$ can be expressed as a linear combination of column vectors of the matrix A, the condition of the existance of the solution (10) is satisfied. The second term in (14) does not disturb the dynamics of the controlled vector $\boldsymbol{w}$. As there is no interaction between $\boldsymbol{w}$ and $\boldsymbol{\theta}$, we find that the control law (14) accomplishes the subobjective. Actually we can derive

$$\begin{aligned} \dot{V}(\boldsymbol{q}) &= (\partial V/\partial \boldsymbol{w})\dot{\boldsymbol{w}} + (\partial V/\partial \boldsymbol{\theta})\dot{\boldsymbol{\theta}} \\ &= (\partial V/\partial \boldsymbol{w})\dot{\boldsymbol{w}} + \boldsymbol{\eta}^T B^{*}A\{\dot{\boldsymbol{w}}_d - K(\boldsymbol{w}-\boldsymbol{w}_d)\} \\ &\quad + \boldsymbol{\eta}^T(I - B^{*}B)\alpha\boldsymbol{\eta}. \end{aligned} \tag{16}$$

As $(I - B^{*}B) \geq 0$ [8], we find that the third term of the input (14) accomplishes the increase of the cost function V.

The closed-loop system is expressed as

$$A\{(\dot{\boldsymbol{w}} - \dot{\boldsymbol{w}}_d) + K(\boldsymbol{w}-\boldsymbol{w}_d)\} = 0. \tag{17}$$

If the matrix A is full column rank, the uniqueness of the solution is guaranteed. The solution of (17) is given as

$$\dot{\boldsymbol{w}} - \dot{\boldsymbol{w}}_d + K(\boldsymbol{w}-\boldsymbol{w}_d) = 0$$

and we find that the controller ensures the convergence of the controlled state vector to the desired value ($\boldsymbol{w} \longrightarrow \boldsymbol{w}_d$). A set of joint angles which satisfies $rank A < q$ (A is not full column rank) means the singular configuration, for example a straight line ($\phi_i = 0, i = 1, \cdots, n-1$).

6. System Design based on Units

Let us introduce the concept of units. We define that a unit is a fundamental element for constructing the redundant snake robot. The serial connection of uniform units constructs a snake robot as shown in Fig. 4.

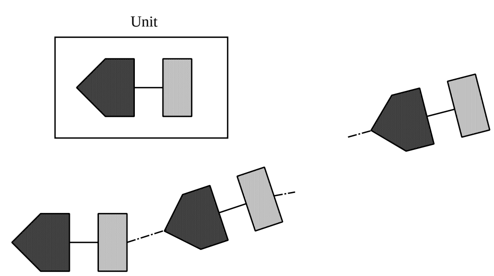


Fig. 4 Concept of unit and total system

6.1. Condition of unit

Let us introduce following [assumption U1] - [assumption U5].

[assumption U1] : A connected joint of two units is passive.

[assumption U2] : The head link of a unit is wheel free.

[assumption U3] : The tail link of a unit is the wheeled link.

[assumption U4] : The link which is introduced the shape controllable point in an unit is wheel free.

[assumption U5] : The passive joint is equivalent to the shape controllable point.

The assumption U1 means that a connection point of units is a free joint. The assumption is acceptable because the actuator can not be mounted on a connected point of units. The assumptions U2-U5 for the unit are related to the assumptions 1-4 for the total system, respectively.

Let n_u be the number of links, m_u be the number of wheeled links, s_u be the shape controllability index of one unit. We assume that same k units are connected serially. Fig. 5 shows an example of the connection of the units. From (12) the condition for the redundancy controllability of the total system of the connected k units can be expressed as

$$\left\{ \begin{array}{lcl} km_u & \geq & 3 + (k-1) + ks_u \\ km_u & < & k(n_u - 1) - ks_u \end{array} \right. . \tag{18}$$

The condition for the unit so that the total system becomes redundancy controllable is given as follows

$$\frac{k+2}{k} + s_u \ \leq \ m_u < \ (n_u - 1) - s_u. \tag{19}$$

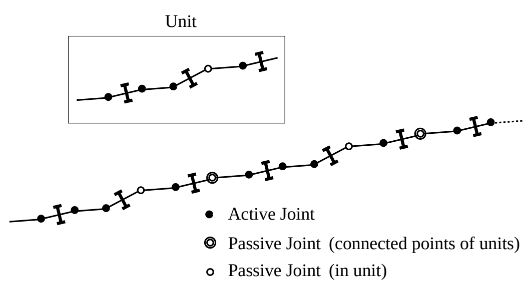


Fig. 5 An example of the connection of the units

We classify the units based on the number of the links n_u and the number of the wheeled links m_u and define $Type(n_u, m_u)$ as the type of units. From the assumptions U2 and U3 we find that the $Type(n_u, m_u)$ unit has ${}_{n_u-2}C_{m_u-1}$ different formations.

Next we discuss the minimum units. Setting $k = 1$ in (19) gives

$$3 + s_u \leq m_u < (n_u - 1) - s_u \tag{20}$$

If we set $s_u = 0$ and 1, from (20) the minimum number m_u, n_u are given as

$$\left\{ \begin{array}{l} s_u = 0: \ \ n_u = 5, m_u = 3 \\ s_u = 1: \ \ n_u = 7, m_u = 4 \end{array} \right. .$$

For $k = 2$ the condition (13) is rewritten as

$$2 + s_u \leq m_u < (n_u - 1) - s_u. \tag{21}$$

If we set $s_u = 0$ and 1, from (21) the minimum number m_u, n_u are given as

$$\left\{ \begin{array}{l} s_u = 0: \ \ n_u = 4, m_u = 2 \\ s_u = 1: \ \ n_u = 6, m_u = 3 \end{array} \right. .$$

We can derive the minimum units as follows:

1. The $Type(4, 2)$ is the minimum unit which can construct the redundancy controllable system by connection.
2. The $Type(6, 3)$ is the minimum unit which can construct the redundancy controllable system by connection and has a shape controllable point in the unit itself.
3. The $Type(5, 3)$ is the minimum unit that the unit itself is the redundancy controllable.
4. The $Type(7, 4)$ is the minimum unit that the unit itself is the redundancy controllable and has a shape controllable point in the unit itself.

From the second inequality of the condition (18) we obtain

$$s_u < n_u - m_u - 1$$

and we find the maximum number of the shape controllable points which can be indroduced in one unit is $n_u - m_u - 2$. If we set $s_u = n_u - m_u - 2 \geq 0$, the first inequality of the condition (18) is rewritten as

$$2m_u + 1 \geq n_u + \frac{2}{k}. \tag{22}$$

Combining $s_u \geq 0$ and (22) yields

$$m_u + 2 \leq n_u < 2m_u + 1. \tag{23}$$

Next we discuss the characteristic of the system which is constructed by connecting the $Type(n_u, m_u)$ units under the assumption that we introduce the maximum number of the shape controllable points in one unit. Let U be the total number of units, p be the total number of the inputs, $q = 3 + s$ be the total number of the states to be controlled, $r = p - q$ be the number of the redundancy, and s_c be the total number of the shape controllability index related to the connected joints of units. As the exclusion of the set of the shape controllable points in an unit means the set of active joints, we find that $p = k\{n_u - 1 - (n_u - m_u - 2)\} = k(m_u + 1)$. We obtain Table 1.

Table 1 Characteristic of the system which is constructed by the $Type(n_u, m_u)$ units

U	n	m	p	q
1	n_u	m_u	m_u+1	$(n_u-m_u-1)+2$
2	$2n_u$	$2m_u$	$2(m_u+1)$	$2(n_u-m_u-1)+2$
3	$3n_u$	$3m_u$	$3(m_u+1)$	$3(n_u-m_u-1)+2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	kn_u	km_u	$k(m_u+1)$	$k(n_u-m_u-1)+2$

U	r	s_c	s_u
1	$2m_u-n_u$	0	n_u-m_u-2
2	$2(2m_u-n_u)+2$	1	$2(n_u-m_u-2)$
3	$3(2m_u-n_u)+4$	2	$3(n_u-m_u-2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	$k(2m_u-n_u)+2(k-1)$	$k-1$	$k(n_u-m_u-2)$

U	s
1	n_u-m_u-2
2	$2(n_u-m_u-2)+1$
3	$3(n_u-m_u-2)+2$
$\vdots$	$\vdots$
k	$k(n_u-m_u-2)+k-1$

From Table 1 we obtain

$$\begin{cases} p = k(m_u+1) \\ r = k(-n_u+2m_u+2)-2 \, . \\ s = k(n_u-m_u-1)-1 \end{cases} \tag{24}$$

Eliminating k in (24) gives

$$r = (2-\frac{n_u}{m_u+1})p-2, \tag{25}$$

$$s = (\frac{n_u}{m_u+1}-1)p-1. \tag{26}$$

From (25) and (26) we obtain the relation

$$r+s+3=p \tag{27}$$

and we find the trade-off of p, r and s.

6.2. System design

We propose the system design strategy of the snake robot based on the units. The design problem is formulated as following.

[**Problem**] : Let n_u, m_u, k, p, r, s be natural numbers.

$$\begin{aligned} Given &: \; p \text{ and } r (\text{or } s) \\ Find &: \; s(\text{or } r), n_u, m_u, k \text{ which satisfy } (24) \end{aligned}$$

The design procedure is as follows :

[**P1**] To give the number p_0 of the input.

[**P2**] To determine the number r_0 of the redundancy and the shape controllability index s_0 under the constraint (27).

[**P3**] To determine the type $Type(n_{u0}, m_{u0})$ and the number k_0 of units.

By using (24) we obtain

$$\frac{n_u}{m_u+1} = \frac{p_0+s_0+1}{p_0} = \frac{2p_0-r_0-2}{p_0} \tag{28}$$

In [P3], we should consider two cases.

(1) p_0+s_0+1 and p_0 are relatively prime

Let us define

$$\begin{aligned} n_{u0} &= p_0+s_0+1 = 2p_0-r_0-2 \\ m_{u0} &= p_0-1. \end{aligned} \tag{29}$$

If n_{u_0} and m_{u_0} satisfy the inequality (23), then the unit is defined as $Type(n_{u_0}, m_{u_0})$ and the number of the units as $k_0=1$. If not, go to [P2].

(2) p_0+s_0+1 and p_0 are not relatively prime

Let a be a common divisor of p_0+s_0+1 and p_0. From the condition $p_0+s_0+1 = an_{u_0}$, $p_0 = a(m_{u_0}+1)$ we obtain

$$\frac{p_0+s_0+1}{p_0} = \frac{an_{u_0}}{a(m_{u_0}+1)}$$

and

$$n_{u_0} = \frac{p_0+s_0+1}{a}, \quad m_{u_0} = \frac{p_0}{a}-1.$$

If n_{u_0} and m_{u_0} satisfy the inequality (23), then the unit is defined as $Type(n_{u_0}, m_{u_0})$ and the number of the units k_0 as

$$k_0 = \frac{p_0}{m_{u_0}+1} = \frac{p_0}{\frac{p_0}{a}} = a.$$

If not, choose another common divisor and take the same procedure (2). If the types for all common divisors do not satisfy the condition, go to [P2].

7. Simulation

To demonstrate the validity of the proposed control law simulations have been carried out. In this simulation we set $B^* = B^T(BB^T)^{-1}$ and

$$V = a'(det(A^TA)) + b'(det(BB^T)) \tag{30}$$

where $a', b' > 0$. The first term of the right side of (30) implies the measure of the singular configuration. The second term of the right side of (30) is related to the manipulability of the system.

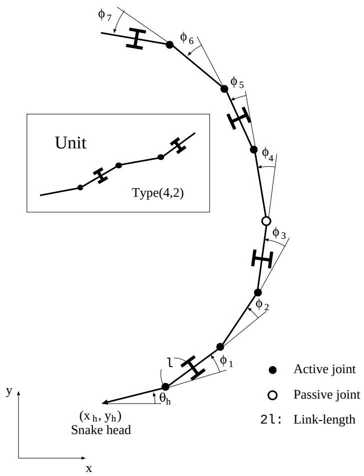


Fig. 6 A 8-link snake robot that is constructed by connecteing two $Type(4,2)$ units

We consider a 8-link snake robot that is constructed by connecteing two $Type(4,2)$ units as shown in Fig. 6. The $Type(4,2)$ unit has four links, two wheeled links and no shape controllable points The connected point is the shape controllable point. In this case $\boldsymbol{w} = [\, x_h \quad y_h \quad \theta_h \quad \phi_4 \,]^T$ and the matrix A is square.

We set the initial condition $\boldsymbol{w}(0), \boldsymbol{\theta}(0)$ and the desired condition $\boldsymbol{w}_d(t)$ as $\boldsymbol{w}(0) = [\, 0 \ 0 \ \frac{3\pi}{4} \ \frac{\pi}{90} \,]^T$, $\boldsymbol{\theta}(0) = [\, \frac{\pi}{120} \ \ \frac{\pi}{110} \ \ \frac{\pi}{100} \ \ \frac{\pi}{80} \ \ \frac{\pi}{70} \ \ \frac{\pi}{60} \,]^T$, $w_d = [\, t \ 0 \ \pi \ \phi_{4_d} \,]^T$, $\dot{w}_d = [\, 1 \ 0 \ 0 \ \dot{\phi}_{4_d} \,]^T$, and $l = 0.05[m]$, $K = diag(3,3,3,3)$. We set cofficients of the cost function V as $a' = a/l^4, b' = b/l^8$ in order to normalize with respect to the link length l. Figs.7-9 show the transient responses. The left column in each figure shows transient responses for $x_h - x_{h_d}[m], y_h - y_{h_d}[m], \theta_h - \theta_{h_d}[m], \phi_4[rad]$, $detA/l^2$, $\sqrt{det(BB^TT)}/l^4$ and the right column shows transient responses for $u_1, \cdots, u_6$. Fig. 10 shows the movement of the snake robot.

Fig. 7 shows the responses for $\alpha = 0, \phi_{4_d} = 0$ ($case$ 1). In this case the controller does not use the redundancy and the desired value for the shape controllable point is zero. From the figure we find that the snake head tracks the desired trajectory, but $detA$ converges to zero. In this case we find that the snake robot converges to a singular configuration of a straight line [9].

Fig. 8 shows the responses for $\alpha = 0, \phi_{4_d} = \frac{\pi}{10}\cos(9t)$ ($case$ 2). In this case the controller does not use the redundancy but the desired value of the shape controllable point is not zero. From the figure we find that the snake head tracks the desired trajectory without converging to the singular configuration and the movement of the snake robot is like *the side winding motion* [1] of snakes (Fig. 10).

Fig. 9 shows the responses for $\alpha = 1, a = 1, b = 0.85, \phi_{4_d} = 0$) ($case$ 3). In this case the controller uses the redundancy but the desired value of the shape controllable point is zero. From the figure we find that the snake head tracks the desired trajectory and the snake robot crawls without converging to the singular configuration (Fig. 10).

From simulation results we find that the second term of the control law (14) can ensure the singularity avoidance and the vibratory motion of the shape controllable point can avoid convergence of the singular configuration.

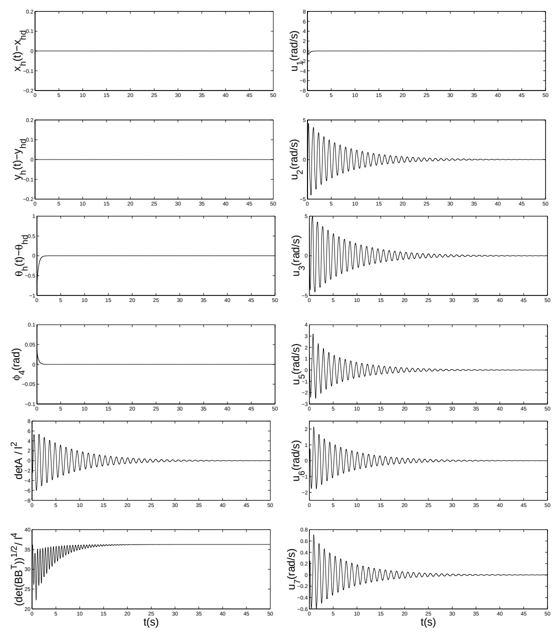
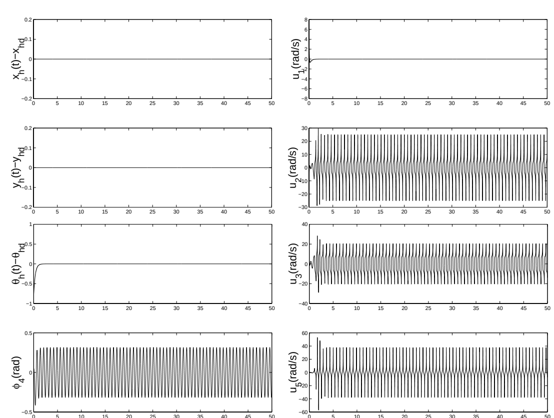


Fig. 7 Transient responses for the controller without considering redundancy ($\alpha = 0,\ \phi_{4_d} = 0$)($case$ 1)



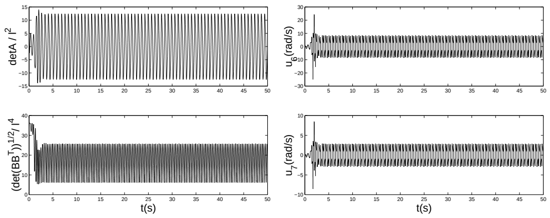


Fig. 8 Transient responses for the controller without considering redundancy
$(\alpha = 0,\ \phi_{4_d} = \frac{\pi}{10}\cos(9t))(case\ 2)$

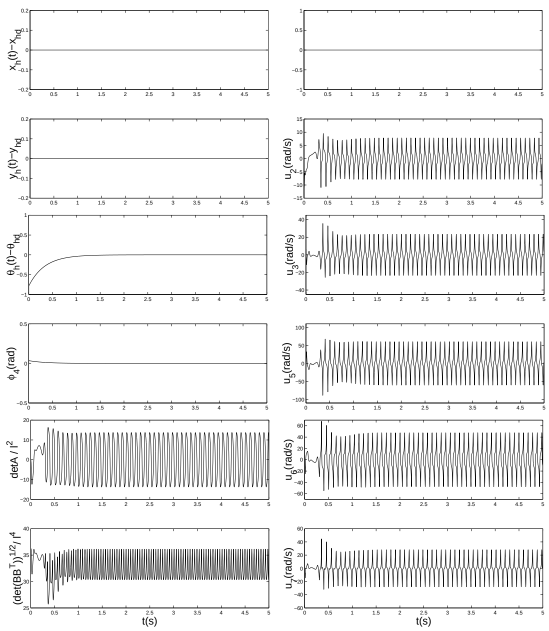


Fig. 9 Transient responses for the controller with considering redundancy
$(\alpha = 1, a = 1, b = 0.85, \phi_{4_d} = 0)(case\ 3)$

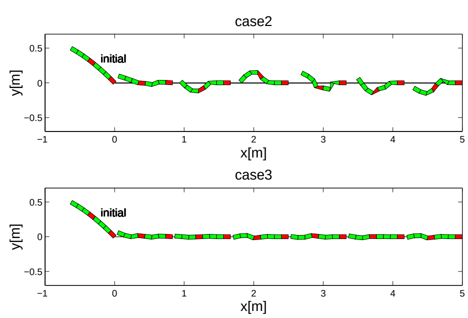


Fig. 10 Movement of the snake robot

8. Conclusion

We derive the condition that the snake robot system becomes redundancy controllable, and introduce the wheel free links in the snake robot body so as to satisfy the condition.

We introduce the concept of the unit and derive the minimum units for several categories. We also propose the system design strategy of the snake robots based on the units.

From simulation results we find that it is possible to accomplish the singular configuration avoidance by giving the appropriate desired value to the shape controllable point or using the redundancy.

As the future works, we should expand the obtained results to the dynamic model.

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