Dynamics and Control of a Simulated 3-D Humanoid Biped

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Abstract

In this paper, we present an algorithm for controlling the walking of a humanoid biped. The algorithm is hierarchical with local Joint Space Control (JSC) and global Virtual Model Control (VMC). VMC plays an important role in the stance phase of the legs and provides postural stability during locomotion. JSC is used at all times, but is essential during the swing phase and for placement of the foot to obtain or change the speed of the robot. A simulation of walking of a three dimensional humanoid biped is achieved through the superposition of these two control schemes.

1. Introduction

Humanoid locomotion is one of the most complex problems to achieve by robots due to mechanical complexity and instabilities. However, solving this problem has many benefits. Humans can locomote over complex terrain that current commercial vehicles can not navigate. A humanoid biped with this locomotion capability could maneuver itself into position to conduct tasks too dangerous for humans. For this purpose, it would be beneficial to have a robot to perform those tasks.

Honda developed a humanoid robot that can walk on a planned path autonomously and perform simple tasks via wireless teleoperation [1]. The pre-defined joint angles of the desired walking pattern are used as reference points for local control of the Honda robot. The desired joint angles are defined via motion studies of human walking. This method is time consuming and not robust. For general locomotion, we can not consider the desired joints angles for JSC to be pre-defined.

In this paper, superposition of Joint Space Control (JSC) and Virtual Model Control (VMC) is used to drive the locomotion of a three dimensional humanoid biped in simulation. For the single support case, the VMC uses an inverted pendulum model. For the double support case this model is modified to generate forces which maintain the center of pressure inside the foot region because of the importance of stability. JSC is implemented using PD Control on desired joint angles obtained from the integration of Cartesian velocities. JSC is used during both stance and swing phases of the legs. Limited walking of the biped is realized by this control model.

To control lateral stability in the frontal plane, J.E. Pratt, and G.A. Pratt [2] use a simple pendulum model for determining the foot placement. Foot placement becomes a function of the velocity in their approach, but velocity control is not seen explicitly. JSC resembles this. On the other hand, JSC has an explicit velocity control component.

2. The Model and Dynamics

Our simulation models the trunk and legs of a human subject and has 18 degrees of freedom. It has two limbs, each with 3 segments, connected to the trunk for a total of 7 segments. The trunk (base body) is defined as a general 6 Degree of Freedom (DOF) body in space. Each limb has a three DOF hip, a one DOF knee, and a 2 DOF ankle.

As shown in Figure 1, each body has a reference frame with its origin located at the proximal joint center. The trunk's reference frame is located at its Center of Mass (COM).

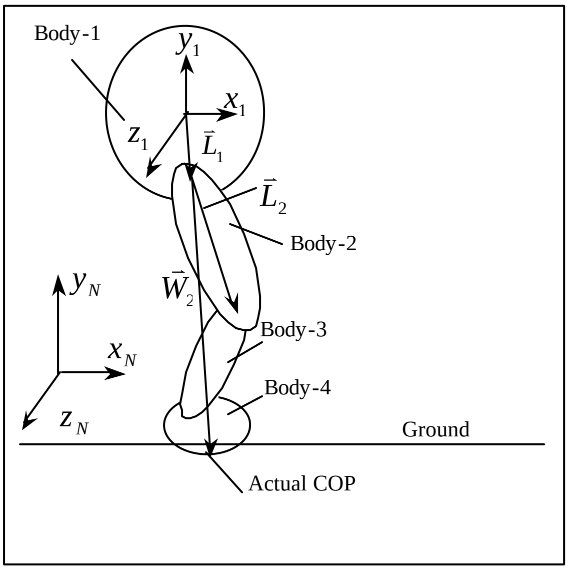

Figure 1: Notation of a Single Leg.

The authors developed a dynamic formulation for a three-dimensional humanoid biped based upon Lagrange's Equations in terms of quasicoordinates [3]. The ground is modeled using stiffness, damping and Coulomb friction. The foot is modeled with many points that can contact the ground. Using these points, the actual center of pressure (COP) can be calculated.

3. Virtual Model Control for the Biped

The posture controller is a global controller based upon VMC and it maintains body stability in all regimes of locomotion, including standing, walking, and running. VMC is presented by Pratt et al. [4]. Contributing research fields are also described well by Pratt [5]. Nelson and Quinn [7] have applied this approach to a cockroach-like hexapod robot. In this paper VMC is applied to control the posture of a three dimensional biped.

3.1. Single limb stance

In the single limb support phase of walking, one leg contacts the ground and provides the force to control the body's motion. The force on the body is produced by pushing on the ground. The limb consists of four links starting from body(1) to foot(4) with body fixed reference frames (1-frame to 4-frame).

The forces at the points on the foot that contact the ground can be resolved into the COP of the foot. The foot can be considered to have a point contact with the ground at the COP and the "unactuated ankle" constraint may be used [7]. Note that the COP is equivalent to the Zero Moment Point (ZMP). This constraint generates a required relationship between the forces and moments applied by the leg, l, on the body 1,

$$\vec{M}_l = \tilde{W}_{1l} C_{1N} \vec{F}_l = \bar{J}_l \vec{F}_l, \qquad l = L, R \tag{1}$$

where $\vec{F}_l$ and $\vec{M}_l$ are these force and moment vectors, respectively. The force vector is expressed with respect to an inertial reference frame (N-frame) while the moment vector is expressed with respect to the body reference frame (1-frame). $\vec{W}_{1l}$ is a position vector of the foot in the body's reference frame, and $\tilde{W}_{1l}$ represents a cross product which is equivalent to a skew operation on this vector:

$$\tilde{W} = \begin{bmatrix} 0 & -W_Z & W_Y \\ W_Z & 0 & -W_X \\ -W_Y & W_X & 0 \end{bmatrix} \tag{2}$$

C_{1N} is a coordinate rotation matrix from the N-frame to the 1-frame. A recursive relationship of $\vec{W}$ can be expressed as follows:

$$\begin{aligned} \vec{W}_1 &= \vec{L}_1 + C_{12}\vec{W}_2 \\ \vec{W}_2 &= \vec{L}_2 + C_{23}\vec{W}_3 \\ \vec{W}_3 &= \vec{L}_3 + C_{34}\vec{W}_4 \\ \vec{W}_4 &= \vec{L}_4 \end{aligned} \tag{3}$$

where $\vec{L}_i$ represents the position vector from the origin of the i-frame to the origin of the (i+1)-frame measured in the i-frame. Thus the transpose of the translational Jacobian for limb 1 with 4 links is expressed as

$$J^T = D^T \begin{bmatrix} \tilde{W}_2^T C_{2N} \\ \tilde{W}_3^T C_{3N} \\ \tilde{W}_4^T C_{4N} \end{bmatrix} \quad (4)$$

where D is a nonlinear function of joint angles.

3.2. Double limb stance

When the right and left legs are both in stance, the relationship between the forces and moments can be expressed as

$$\begin{bmatrix} F \\ M' \end{bmatrix} = \begin{bmatrix} I & I \\ \tilde{W}_1^R C_{1N} & \tilde{W}_1^L C_{1N} \end{bmatrix} \begin{bmatrix} F_R \\ F_L \end{bmatrix}$$

or

$$\begin{bmatrix} F \\ M \end{bmatrix} = \begin{bmatrix} I & I \\ \tilde{p}_f^R & \tilde{p}_f^L \end{bmatrix} \begin{bmatrix} F_R \\ F_L \end{bmatrix} = A \begin{bmatrix} F_R \\ F_L \end{bmatrix} \quad (5a,b)$$

where

$$\tilde{p}_f = C_{N1} \tilde{W}_1 C_{1N} = Skew\left[C_{N1} \vec{W}_1\right]$$

Where F and M are vectors representing the total forces and moments on the body from the stance legs. While M' is represented in the body frame and M is represented in the inertial reference frame, they represent the same vector. $\tilde{p}_f$ is the skew symmetric matrix of the position vector of the foot's contact point with respect to the body in the inertial reference frame. The subscripts R and L represent the right and left feet and I is a 3x3 identity matrix. The matrix A is defined by Eq. (5b).

3.2.1. Solving for the Force Distribution

Given F and M from the VMC, the forces on the right and left legs, F_R and F_L, need to be resolved to distribute the load between the legs. Attempted inversion of the matrix A in Eq. (5b) yields a singularity, because the third and sixth rows of the matrix A are linearly dependent. To solve this under constrained problem we use an optimization. To simplify the problem, we introduce dimensionless parameters that represent the lateral forces and the vertical force as follows:

$$c_{xl} = \frac{F_{xl}}{F_{yl}}, \quad d_{zl} = \frac{F_{zl}}{F_{yl}} \quad (6)$$

$$n_l = \frac{F_{yl}}{F_y}, \quad \sum n_l = 1, \quad l = R, L \quad (7a,b)$$

Introducing Eqs. (6) and (7a) into the six scalar equations represented by Eq. (5b), we have 5 equations, two of them being linearly dependent, and Eq. (7b). When the COP is expressed as

$$\begin{bmatrix} x_{cp} \\ z_{cp} \end{bmatrix} = \begin{bmatrix} p_{xR} & p_{xL} \\ p_{zR} & p_{zL} \end{bmatrix} \begin{bmatrix} n_R \\ n_L \end{bmatrix} \quad (8)$$

then, those linearly dependent equations yield [7]

$$x_{cp} = \frac{M_z}{F_y} + y\frac{F_x}{F_y}, \quad z_{cp} = -\frac{M_x}{F_y} + y\frac{F_z}{F_y} \quad (9a,b)$$

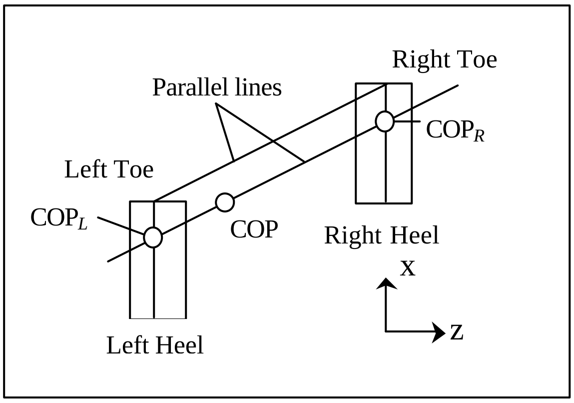

Figure 2: The COP Distribution for the Double Stance

Figure 2 represents the double stance case showing the common COP and distribution of the COP to the right and left legs, COP_L and COP_R. The total biped COP location corresponds to a single leg model of the biped on a flat surface. Note that y is the average of P_L and P_R. For the given F and M, x_{cp} and z_{cp} can be solved from Eq. (9a,b). The COP for double stance is projected to the right and left feet by drawing a line parallel to the line of the right and left toes from the COP as shown in Figure 2. This determines COP_L and COP_R. Now, we can solve Eq. (8) for the vertical weight distribution. This solution has a singularity when the COP's of the individual feet are symmetric about the origin of the trunk-frame. This is avoided by simply enforcing the sum of the vertical force coefficients to be equal to one when this singularity occurs. Now, we are left with three equations and four unknowns to complete the solution of the force distribution problem. We could add an additional constraint [4] or use an optimization [7]. We use optimization to solve the problem with the following cost function

$$E = \frac{1}{2}[\left(c_{xR} - \bar{c}_{xR}\right)^2 + \left(c_{xL} - \bar{c}_{xL}\right)^2 + \left(d_{xR} - \bar{d}_{xR}\right)^2 + \left(d_{xL} - \bar{d}_{xL}\right)^2] \quad (10)$$

where symbols with the over score represent the desired dimensionless force directions. Minimizing E, using the above three linearly independent equations from Eq. (5b) as constraints, solves the force distribution problem completely and encourages each leg to push in a preferred direction.

3.3. Virtual Forces and Moments

An inverted pendulum model is used to find the desired virtual forces and moments that produce a stable posture. In this model a spring and damper in parallel are connected to the origin of the trunk from the COP with length L. The desired virtual force in the direction of L can be computed as

$$F = K_p(L_d - L) + K_v(\dot{L}_d - \dot{L}) \quad (11)$$

where L_d is the desired length of one limb and K_p , and K_v are the proportional and derivative gains. For the single stance phase, L_d is defined as the height of origin of the trunk when the robot is standing up straight. For the double support phase L_d is calculated from a desired moving point that satisfies the initial condition of the inverted pendulum mode for the next single stance leg. F can be projected into the x, y and z directions to obtain the force components.

The desired torques acting on the main body can also be expressed as

$$\tau_{ti} = K_{pi}(\alpha_{di} - \alpha_i) + K_{vi}(\dot{\alpha}_{di} - \dot{\alpha}_i), \quad i = 1,2,3 \quad (12)$$

where K_p , and K_v are the proportional and derivative gains. α , and α_d are the trunk's actual and desired orientations. The $\vec{M} = C_{N1} D_1^{-T} \vec{\tau}_t$ relation is used to transform τ_{ti} to $\vec{M}$.

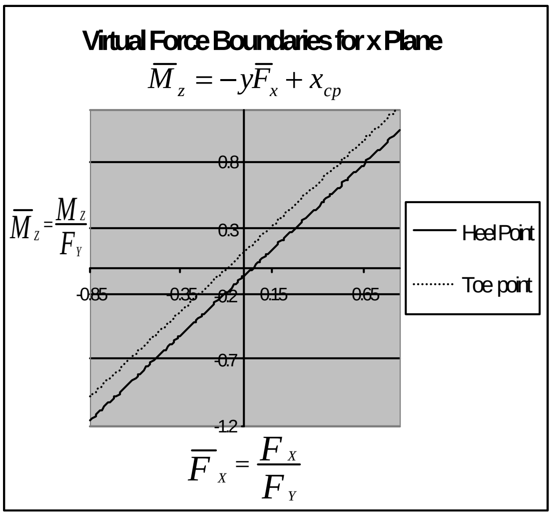

Figure 3: Virtual Force Boundaries for the xz Plane, vertical axis in meters, and horizontal axis in dimensionless units.

Using these sets of forces and moments, the desired COP can be calculated from Eq. (9a,b). The result may be outside of the foot region and therefore impossible to realize.

This led us to develop a scheme that changes the sets of desired virtual forces to realizable sets of forces and moments. In Figure 3, Eq. (9a) is transformed into a plot of virtual force boundaries for the xz plane. The region between the two lines represents the realizable sets of virtual force F_x and moment M_z. When the desired virtual force and moment fall outside of this region, they are modified to be brought into the boundaries at the closest point. The modified force and moment are the input force and moment for the VMC. Similarly, Eq. (9b) is evaluated in order to get a realizable set of F_z and M_z. The above applies to both single stance and double stance in a similar manner.

4. Joint Space Control

JSC is responsible for the swing phase and for placement of the foot at the end of the swing phase. In order to implement JSC using proportional control, we need to obtain some desired joint angles. In this respect, we will introduce a Jacobian that maps the foot cartesian velocity with respect to the trunk to the joint speeds. Let us express the angular velocity of the ground as follows:

$$\vec{w}_g = C_{N1}D_1\dot{\boldsymbol{a}}_1 + C_{N2}D_2\dot{\boldsymbol{a}}_2 + C_{N3}D_3\dot{\boldsymbol{a}}_3 + C_{N4}D_4\dot{\boldsymbol{a}}_4 + \vec{\Omega}_g = 0 \quad (13)$$

which is zero because the ground is fixed. $\vec{\Omega}_g$ is the angular velocity of the ground with respect to the foot. D_i is a nonlinear function of the i^{th} joint angles $\boldsymbol{a}_i$ Solving for the trunk angular speeds yields

$$D_1\dot{\vec{a}}_1 + \vec{\Omega}_g = \begin{bmatrix} -C_{12}D_2 & -C_{13}D_3 & -C_{14}D_4 \end{bmatrix} \begin{bmatrix} \dot{\vec{a}}_2 \\ \dot{\vec{a}}_3 \\ \dot{\vec{a}}_4 \end{bmatrix}$$

or (14a,b)

$$D_1\dot{\boldsymbol{a}}_1 + \vec{\Omega}_g = J_{fa} \begin{bmatrix} \dot{\boldsymbol{a}}_2 \\ \dot{\boldsymbol{a}}_3 \\ \dot{\boldsymbol{a}}_4 \end{bmatrix}$$

where J_{fa} is a rotational Jacobian (3x6) matrix.

Furthermore, the foot velocity with respect to the trunk can be expressed as

$$\dot{\vec{p}}_f = \begin{bmatrix} \tilde{L}_1^T & C_{12}\tilde{L}_2^T & C_{13}\tilde{L}_3^T & C_{14}\tilde{L}_4^T \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ C_{21} & I & 0 & 0 \\ C_{31} & C_{32} & I & 0 \\ C_{41} & C_{42} & C_{43} & I \end{bmatrix} \begin{bmatrix} D_1 & 0 & 0 & 0 \\ 0 & D_2 & 0 & 0 \\ 0 & 0 & D_3 & 0 \\ 0 & 0 & 0 & D_4 \end{bmatrix} \begin{bmatrix} \dot{\vec{a}}_1 \\ \dot{\vec{a}}_2 \\ \dot{\vec{a}}_3 \\ \dot{\vec{a}}_4 \end{bmatrix}$$

or

$$\dot{\vec{p}}_f - \tilde{W}_1^T D_1\dot{\vec{a}}_1 = \begin{bmatrix} C_{12}\tilde{W}_2^T D_2 & C_{13}\tilde{W}_3^T D_3 & C_{14}\tilde{W}_4^T D_4 \end{bmatrix} \begin{bmatrix} \dot{\vec{a}}_2 \\ \dot{\vec{a}}_3 \\ \dot{\vec{a}}_4 \end{bmatrix}$$

or (15a,b,c)

$$\dot{\vec{p}}_f - \tilde{W}_1^T D_1\dot{\boldsymbol{a}}_1 = J_{ft} \begin{bmatrix} \dot{\boldsymbol{a}}_2 \\ \dot{\boldsymbol{a}}_3 \\ \dot{\boldsymbol{a}}_4 \end{bmatrix}$$

where J_{ft} is 3x6 Jacobian matrix. Combining Eq. (14b) and (15c), yields

$$\begin{bmatrix} D_1\dot{\boldsymbol{a}}_1 + \vec{\Omega}_g \\ \dot{\vec{p}}_f - \tilde{W}_1^T D_1\dot{\boldsymbol{a}}_1 \end{bmatrix} = \begin{bmatrix} J_{fa} \\ J_{ft} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{a}}_2 \\ \dot{\boldsymbol{a}}_3 \\ \dot{\boldsymbol{a}}_4 \end{bmatrix}$$

or (16a,b)

$$b = J_f \dot{\boldsymbol{a}}_c$$

where J_f is a 6x6 Jacobian matrix. By inverting J_f, we can obtain the joint angular velocities as functions of b, then we can integrate to find joint angles. When we prescribe b, a desired set of joint angles and velocities can be determined. These are equilibrium points for PD control. The following choices for the b vector are used in our controller:

$$b_{st} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -v_x \\ -v_y \\ -v_z \end{bmatrix}, \quad b_{sw} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ v_x + K_{\dot{x}}(v_{xbsw} - v_{xd})/T \\ BCos(\pi t/T) \\ v_z + K_{\dot{z}}(v_{zbsw} - v_{zd})/T \end{bmatrix} \tag{17}$$

where $v_x, v_y, and\, v_z$ are the instantaneous velocities of the trunk in the x, y, and z directions. Also, T is the desired period of the swing phase, *B* is a constant for the swing leg's vertical travel, t is time, and *st* and *sw* represent the stance and swing legs, respectively. The additional terms $K_{\dot{x}}(v_{xbsw} - v_{xd})$, and $K_{\dot{z}}(v_{zbsw} - v_{zd})$ for the velocity control in the fourth and sixth rows of b_{sw} need to be explained in more detail. These terms are used by Raibert [8] as velocity control components for hopping robots. $K_{\dot{x}}, and\, K_{\dot{z}}$ are feedback gains. $v_{xd}, and\, v_{zd}$ are desired trunk velocities in the x and z directions. $v_{xbsw}, and\, v_{zbsw}$ are the trunk velocities at the beginning of the swing phase, however, previous average velocities can also be used. From simulation results, it was found that this solution produces a bended knee configuration at the end of the swing phase. In order to straighten the knee joint for transition to stance, we forced the knee to open during the swing phase. $\vec{\Omega}_g$ and $\dot{\alpha}_1$ are set to zero.

5. Simulation

A simulation of standing, and some trunk motion was performed using the posture controller based on VMC. The results are stable. A walking simulation of the biped robot was also conducted by combining VMC and JSC. In the forward (x) direction, a stable walking was observed. However, in the side-to-side (z) direction an instability occurs after several steps. We believe that the system can be made to be stable through the right choice of the *b* vector component in the lateral direction. In Figs. 4, and 5, measured COP, and desired COP

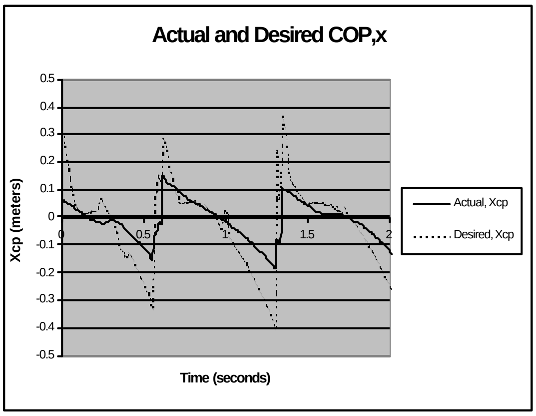

Figure 4: Actual and Desired COP of the Biped in the x direction.

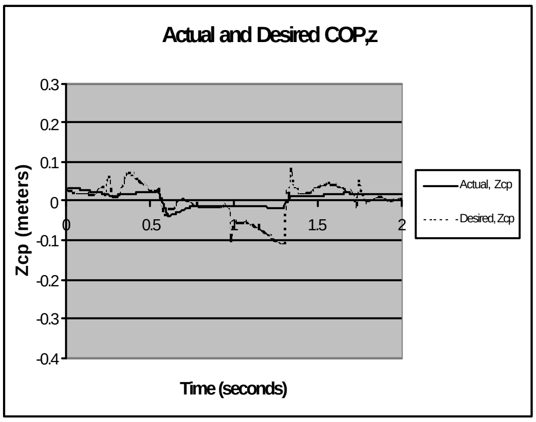

Figure 5: Actual and Desired COP of the Biped in the z direction.

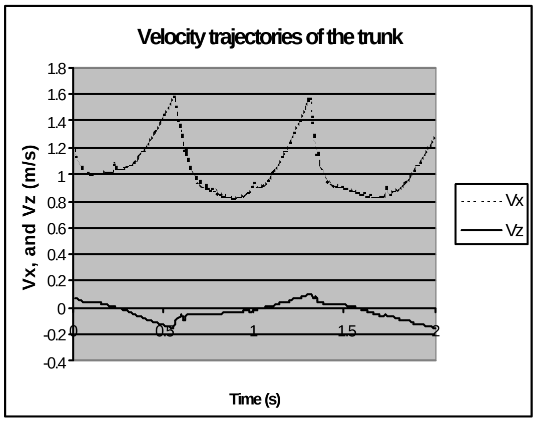

Figure 6: Velocity Trajectories of the Trunk.

with respect to the trunk are plotted versus time. Velocity trajectories in the x and z directions are shown in Fig. 6.

6. Conclusions

In legged locomotion, posture control plays a central role for stability. This is particularly important for a humanoid biped which is statically unstable. When its stability is disturbed, such as by a push, the robot may need to take a step to prevent it from falling. Therefore, foot placement by a swing leg on the ground must be chosen properly. These concepts can be realized by implementing VMC and JSC schemes.

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