Approximate Solutions for Gait Simulation and Control.

Paul Bourassa(1), Margrit-R. Meier(1), Philippe Micheau(2), Paulin Buaka(2)

Université de Sherbrooke, Qué, Canada

(1) IUGG, centre de recherches.

(2) Faculté de génie

A biped model is analysed and its relevant second order differential equation of motion are obtained using the Lagrange or the Kane formalism. Previous studies by McGeer [1], Goswani & al. [2], Garcia & al. [3], and others have been very helpful in the understanding of bipedal passive locomotion under gravity alone. This is an attempt to go somewhat further in the same direction: a biped moving on a level surface under the influence of impulsive forces. A four d.o.f. model is studied in the sagittal plane over a visco-elastic medium. A two d.o.f. model with one foot fixed to a rigid floor is also investigated in order to get closed form results. With proper initial conditions, impulses and torques, the programme is able to numerically provide a solution, which lasts for many consecutive steps, leading to steady and stable limit cycles in angular velocities. An animation programme has been proven useful in showing such sequences. The problem addressed here is how to compute the value of the feet impulses and the corresponding torque amplitude in order to attain a given gait velocity and maintain a repeatable gait pattern.

Model description: The model is shown in Figure 1. It consists of two rigid segments representing the left and the right leg with mass M_A located at mid length and a central moment of inertia $I_{A.}$ The trunk is assumed as a point mass M_e located at E. Figure 1 shows the right leg resting on the floor while the left is subject to a short impulse directed along the segment. Afterward, an internal torque T_{BA} is applied to the left leg. A similar reversed scenario occurs on the following step, with a torque Tab applied to left leg The Lagrange equations for the two d.o.f model walking on a rigid surface are represented in Equation (1):

$$[A(q_i)]\begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} f_1(q_i,U_i) \\ f_2(q_i,U_i) \end{Bmatrix} + \begin{Bmatrix} c_1 F_{i1} \\ c_2 F_{i2} \end{Bmatrix} \qquad (1)$$

where A is a 2x2 matrix; q_i and U_i, are the state variables; (i = 1,2), F_{i1}, and F_{i2} are the required impulses for either leg; U_1 and U_2 are the time derivatives of q_1 and q_2. The matrix A consists of four elements a_{ij} with

$a_{11} = (I / ML^2 + 1/4)$ $\qquad a_{12} = -\cos(q_1-q_2) / 2$

$a_{21} = a_{12}$ $\qquad a_{22} = (I/ML^2 + 5/4 + M_e/M)$

$f_1(q_i,U_i) = 0.5 \sin(q_1-q_2)U_2^2 + 0.5\, G/L \cos(q_1) + T_{ba}/ML^2$

$f_2(q_i,U_i) = - 0.5 \sin(q_1-q_2)U_1^2 + (M_e/M + 3/2)\, G/L \cos(q_2) - T_{ba}/ML^2$

$c_1 = \sin(q_1-q_2)/ML$ or 0 $\qquad c_2 = \sin(q_1-q_2)/ML$ or 0

The determinant. $\det(A) = a_{11}a_{22}-a_{12}\, a_{21}$ and $a_{12} = a_{21}$ has one term a_{12} which is very slightly co-ordinate dependant.

G is the constant of gravity, assumed to be $10 m.s^{-2}$. Similar expressions are obtained when interchanging the angular leg generalised co-ordinates q_1 and q_2, and using T_{ab} in place of T_{ba}, when the left leg is in the support mode and the right leg is in the swinging mode. These equations are used to derive some basic relationships between the torque, the impulse and the kinematics of the walking gait pattern. The system is solved numerically using Runge-Kutta algorithm. The code is provided with branching in order to generate a set of multi-step sequences. Ground interference with the swinging leg is ignored except at contact time where ground reaction forces are evaluated. Such a system has the advantages of producing continuous motion from one step to another by simple branching and constants resetting, as the model goes from one configuration to the next.

Impulse: During the push-off phase in human gait, the foot reacts against the ground to produce a reaction force. Its integral over time is known as an impulse. During that phase, the legs are changing angular velocities and the body is rising. The simplest form of an impulse is a force amplitude F_{imp} applied over a single time step. The integration of (1) over a short impulse time results in a sudden increase in the angular velocities of all the generalized velocities of the system. For the present model the relationship is given in Equation (2).

$$\Delta U_1 = f_3(q_i, U_i)\Delta t + f_4 \Delta U2 \qquad (2)$$
$$\Delta U_2 = f_5(q_i, U_i)\Delta t + f_6(F_{i2}\Delta t)$$

The angular velocity increments are seen to be function of the impulse value. Thereafter, a properly selected torque T_{ba} will accelerate and bring the swinging leg to an eventual maximum velocity v_m and in phase with the support leg at mid-stance. To this end, we investigate the equations [1] with a null value for the impulse and find out that f_1 in the second equation is relatively small. Considering furthermore small angles approximations, the expression for the derivative dU_2/dt becomes

$$dU_2/dt = a(q_1 - q_2)U_1^2 + b(\pi/2 - q_2) + cT_{BA} \qquad (3)$$

U_1 can be shown to be increasing in a nearly linear manner with time and to be proportional to T_{BA}. Therefore the above equation takes the form

$$d^2q_2/dt^2 = c_1 + c_2 q_2 + g(t) + c_3 t^2 q_2 \qquad (4)$$

This equation is linear and a power solution for q_2 may be found by the method of variable coefficients. From this solution, the time t_{MS} required to reach mid-stance ($q_2=\pi/2$) and the velocity at mid-stance $U_2(t_{MS})$ may be obtained. These values are functions of T_{ba} and of the values U_{1+} and U_{2+} immediately following the impulse. These U_i values have just been given angular velocities increments which are fonction of the impulse intensity. Therefore it is possible to establish a required initial impulse and a proper torque amplitude that will yield a synchronised stance for a given velocity at mid-stance. Examples of such solutions and corresponding animations will be shown in the presentation. After applying the initial impulse, F_{img} is taken as zero until the next step occurs and equation (1) is than easily inverted. The synchronised mid-stance angles and velocity objectives may also be expressed by algebraic equations which may be solved numerically by Newton Raphson algorithm and also by repeated integration.

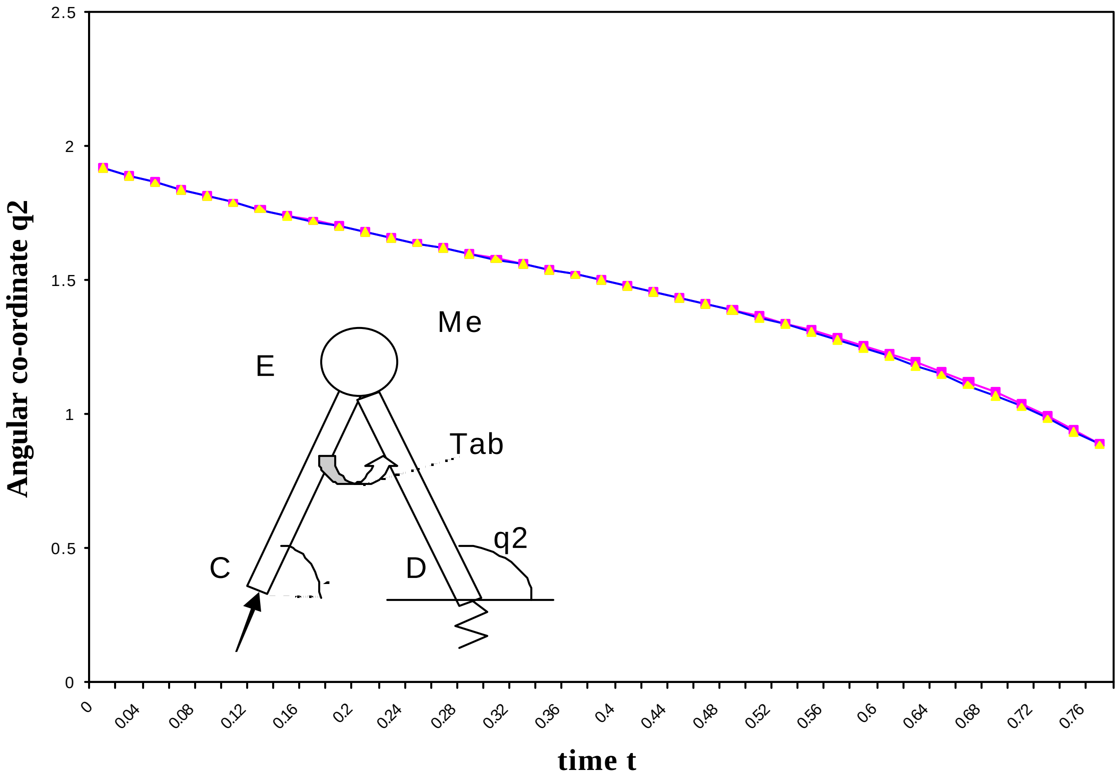
Polynomial solution: The angles q_1 and q_2 range from 70 to 110 degrees approximately and approximations may be taken for linearization purposes. Replacing in equation (1) sin(q1-q2) by (q1-q2) , cos(q1-q2) by 1, det(A) becomes a constant and the expressions on the right hand side take the following form:

$$f_1 = 0.5(q_1-q_2)\, U_2^2 + G/2L + T_{ba}/ML^2$$
$$f_2 = -.5(q_1-q_2)\, U_1^2 - (M_e/M+3/2)\, G/2L - T_{ba}/ML^2 \qquad (6)$$

The torque T_{ba} will be taken as a constant during the first part of the walking step until mid-stance. After mid-stance, a change of sign for T_{ba} will be introduced in the simulation that will slow down the fast moving leg. We may rewrite the system as

$$dU1/dt = [(a11.f1g - a12.f2g) + (a22+a12)T'_{ab}]/detA$$
$$dU2/dt = [(-a12f1g+a11f2g) - (a11+a12)T'_{ab}]/detA \qquad (7)$$

Figure 1: Polynomial Solution q2p(t) vs integrated solution q2(t)



The equations may be uncoupled when observing during simulation that the term including the torque T_{ba} in the first differential equation is dominant. This results into $dU_1/dt = T_{ba}/I_e$ where $I_e = I_a + M L^2$ and represents the leg moment of inertia around the hip pivot point. Therefore $U_1(t) = U_1(0) + (T_{ba}/I_e)t$ and $q_1(t)=q_1(0) + U_1(0)t + 0.5(T_{ba}/I_e)t^2$. This simple solution has been verified with sufficient accuracy for the cases at hand. The quadratic form may be solved to find out the time t_{1ms} at which the swinging leg will reach a vertical position.

$$t_{1ms} = -b + \sqrt{(b^2-c)} \quad \text{where } b=(I_e/T_{ba})U_1(0) \text{ and } c = [(2q_1(0) - \pi/2]\, I_e/T_{ba} \qquad (8)$$

where the plus sign must be assumed in front of the square root. In the second differential equation, the term $a_{12}f_{1g}$ is small compared to the others terms and the replacement of $U_1(t)$ produces a time dependent linearized equation of the form

$$d^2q_2/dt^2 = dU_2/dt = e_0 + e_1t + e_2t^2 + e_3t^3 + e_4t^4 + (e_5 + e_6t + e_7t^2)q_2 \qquad (9)$$

where the expressions for e_0 to e_7 are given in the appendix. This linearized, time-dependant differential equation may be solved using a polynomial solution for q_2 in powers of t.

$$q_2 = b_0 + b_1t + b_2t^2 + \ldots\ldots + b_nt^n \quad \text{and}$$
$$U_2 = b_1 + 2b_2t + 3b_3t^2 + \ldots.. + (n+1)b_{n+1}t^n \qquad (10)$$
$$dU_2/dt^2 = 1 \circledR 2b_2 + 2 \circledR 3b_3t + 3 \circledR 4b_4t^2 + \ldots..(n+1)*(n+2)b_{n+2}t^n$$

The replacement of these polynomial into the differential equation yields a polynomial in t^i. Each coefficient must be zero to give the recurrent expressions for the b_i coefficients in terms of the initial conditions and in terms of the known e_i coefficients. These are as follows:

$$b_o = q_2(0) \quad b_1 = U_2(0) \quad b_2 = (e_0 + e_5b_o)/2 \quad b_3 = (e_1 + e_6b_o + e_5b_1)/2 \circledR 3$$
$$b_4 = (e_2 + e_7b_o + e_6b_1 + e_5b_2)/3 \circledR 4 \quad b_5 = (e_3 + e_7b_1 + e_6b_2 + e_5b_3)/4 \circledR 5 \qquad (11)$$
$$b_6 = (e_4 + e_7b_2 + e_6b_3 + e_5b_4)/5 \circledR 6 \quad b_7 = (e_7b_3 + e_6b_4 + e_5b_5)/6 \circledR 7$$
$$b_{n+2} = (e_7b_{n-2} + e_6b_{n-1} + e_5b_n)/(n+1) \circledR (n+2)$$

This polynomial solution for the leg angular co-ordinate and for its angular velocity has shown to be a very good approximation for the integrated numerical solution using Runge-Kutta algorithm, for cases where the balancing leg is reaching a vertical position at approximately the same time as the supporting leg Figure 2).

Discussion and conclusion: The generated gait cycles comprise a multitude of continuous steps accompanied by steady and stable limit cycles in angular velocities. The relationships between the impulse, the torque, the model parameters and initial conditions pave the way to a better understanding and simulation. Animation is found to be a very useful tool in order to assess the progress made during the whole development. Further refinements are required in order to broaden the stability range of successful gait solutions. An approximate solution to the linearized set of differential equations provide some important relationships between the model variables at impulse time, its parameters and the values of the impulse and of the constant torque in order to achieve a continuous and repeatable gait cycle with a desired mid-stance velocity.

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References:

1 McGeer, T (1990). Passive dynamic walking. *Int J Robotic Res* 9:62-82.
2 Goswani A, Thuilot B and Espiau B (1998). A study of the passive gait of a compass-like biped robot: symmetry and chaos. *Int J Robotic Res* 17(12): 1282-1301.
3 Garcia M, Chatterjee A, Ruina A, Coleman M (1998) The simplest walking model: stability, complexity, and ccaling" *J Biomech Eng* 120: 281-287.
4 Bourassa P et al. Gait Modelling, Animation and Limit Cycle, 7th ISCSB, Calgary 1999